Advanced Composite Materials
Vol. 21, Nos. 5–6, October–December 2012, 477–489

Skin effect of alternating electric current on laminated CFRP

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(Received 2 June 2012; accepted 28 September 2012)

When an alternating current is applied to a conductive slab, the induced current impedes the electric current and causes a skin effect. This effect is found in all conductive materials. For laminated carbon fiber reinforced plastics (CFRP), however, the depth of the skin effect has not previously been evaluated. In the present study, therefore, we analytically solve Maxwell's equations to derive the skin effect depth of unidirectional CFRP. Using this result, the skin effect is analyzed for laminated CFRP, and its depth is then derived. The effect is then compared with a newly defined skin effect of anisotropic conductance. For highly toughened CFRP that have resin rich layers, the skin effect of anisotropic conductance is found to be more important than that of the alternating current.

Keywords: laminated composites; electric conductance; skin effect; Maxwell's equations

1. Introduction

Laminated carbon fiber reinforced plastic (CFRP) composites have been widely adopted for primary aircraft structures. Damage caused by lightning strikes and leakage of electric current to fuel tanks are significant issues for such structures. Leaked electric current may ionize the matrix resin in such composites, causing a thermal spark in the fuel tank. Several studies have reported on resin damage after lightning strikes [1–3]. On actual aircraft, copper mesh is installed as lightning-strike protection. Experimental investigations have been performed on the effect of leaked or induced electric current from these lightning protection systems. The author has proposed a simple analytical method for calculation of electric current for thick laminated CFRP composites, using potential perfect fluid flow analysis [4,5]. This new analytical method is very effective even for low-frequency alternating currents, which can be approximately treated as a steady flow. For high-frequency alternating currents, however, the skin effect caused by the induced current arising from the alternating electromagnetic field must be considered. This leaves the limitations of this potential flow analytical method unclear.

When alternating current flows in a conductive material, the resulting alternating electromagnetic field induces an electric current in the opposite direction. The induced electric current reduces the electric current flow inside the conductive material and limits it to the skin area near the surface. This is known as the skin effect of alternating current. Use of laminated conductive materials to reduce the skin effect problem has been proposed in some reports

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ISSN 0924-3046 print/ISSN 1568-5519 online

http://dx.doi.org/10.1080/09243046.2012.743716
http://www.tandfonline.com

[6,7]. However, the skin effect in laminated CFRP, which has strong orthotropic conductance, has not yet been investigated. In the research on eddy current inspection of CFRP laminates, skin effect depth was calculated using an equation created for isotropic metallic materials [8].

In the present study, therefore, the skin effect depth of orthotropic-conductive CFRP laminates is investigated. Maxwell's equations are used for the analysis, and the skin effect theory is extended to CFRP laminates. The skin effect depth of alternating current in CFRP laminates is derived, and the effect is compared with that of the concentration of electric current caused by orthotropic conductance through analysis of the direct electric current.

2. Maxwell's equations

Maxwell's equations govern the electromagnetic field of an alternating electric current. The Maxwell's equations of orthotropic materials are given as follows. To show the orthotropic electric properties clearly, the vector-operation arithmetic indicator is not used here. In the following, the fiber direction is the x-coordinate, transverse direction is the y-coordinate, and thickness direction is the z-coordinate.

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} + \frac{\partial B_x}{\partial t} = 0 \tag{1}$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} + \frac{\partial B_y}{\partial t} = 0 \tag{2}$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} + \frac{\partial B_z}{\partial t} = 0 \tag{3}$$

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} - \frac{\partial D_x}{\partial t} = i_x \tag{4}$$

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} - \frac{\partial D_y}{\partial t} = i_y \tag{5}$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} - \frac{\partial D_z}{\partial t} = i_z \tag{6}$$

$$\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = \rho \tag{7}$$

$$\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0 \tag{8}$$

$$B_x = \mu_x H_x, \ B_y = \mu_y H_y, \ B_z = \mu_z H_z \tag{9}$$

$$D_x = \varepsilon_x E_x, \ D_y = \varepsilon_y E_y, \ D_z = \varepsilon_z E_z \tag{10}$$

where, B is magnetic flux density, H is magnetic field, D is electrical flux density, E is electric field, μ is magnetic permeability, ε is dielectric constant, i is electric current density, and ρ is electric charge.

The wavelength λ of an electromagnetic wave can be obtained from the frequency f of the wave and the speed of light c.

$$\lambda = \frac{c}{f} \approx 3.0 \times 10^8/f \tag{11}$$

When the wavelength λ is large enough compared with the thickness h of the CFRP plate, the displacement current $\partial D/\partial t$ can be neglected in the thickness direction distribution of electric current flow [6]. The thickness of CFRP plate for aircraft components is approximately 30 mm at the maximum, and the spacing between any two metallic parts, such as rivets or bolts, is approximately 3 m. When a factor of 100 is adopted as a sufficiently large distance compared with the wavelength, Equation (11) indicates that a frequency of 100 MHz can be assumed to be a quasi-steady current for this thickness; the displacement current $\partial D/\partial t$ is negligible in the thickness direction. For the in-plane direction, a frequency of 1 MHz is assumed to be a quasi-steady current. Equations (4–6) then become as follows:

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = i_x \tag{12}$$

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = i_y \tag{13}$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = i_z \tag{14}$$

Electric current density is proportional to electric field for the orthotropic conductance material.

$$i_x = \sigma_x E_x, \ \ i_y = \sigma_y E_y, \ \ i_z = \sigma_z E_z \tag{15}$$

This is called a quasi-steady current. In Equation (15), σ_x, σ_y, and σ_z are electric conductance in the x-direction, y-direction, and z-direction, respectively. When Equation (15) is substituted into Equations (12–14) and Equation (9) is substituted into Equations (1–3), the equations of the quasi-steady current are obtained as follows:

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -\mu_x \frac{\partial H_x}{\partial t} \tag{16}$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -\mu_y \frac{\partial H_y}{\partial t} \tag{17}$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -\mu_z \frac{\partial H_z}{\partial t} \tag{18}$$

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = \sigma_x E_x \tag{19}$$

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = \sigma_y E_y \tag{20}$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = \sigma_z E_z \tag{21}$$

Because CFRP composites are composed of nonmagnetic materials, the relative magnetic permeability of the CFRP is set to 1; the value is isotropic and the same as that of the magnetic permeability constant of a vacuum. Differentiating Equation (19) with respect to t and substituting Equations (17) and (18) to remove H_z and H_y produces the following equation:

$$\begin{aligned} -\mu\sigma_x\frac{\partial E_x}{\partial t} &= \left(\frac{\partial^2 E_y}{\partial x\partial y} - \frac{\partial^2 E_x}{\partial y^2}\right) - \left(\frac{\partial^2 E_x}{\partial z^2} - \frac{\partial^2 E_z}{\partial x\partial z}\right) \\ &= \left(\frac{\partial^2 E_y}{\partial x\partial y} + \frac{\partial^2 E_z}{\partial x\partial z}\right) - \left(\frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2}\right) \\ &= \frac{\partial}{\partial x}\left(\frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}\right) - \left(\frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2}\right) \end{aligned} \tag{22}$$

From Equation (7), having no electric charge in the conductive material gives:

$$\varepsilon_x\frac{\partial E_x}{\partial x} + \varepsilon_y\frac{\partial E_y}{\partial y} + \varepsilon_z\frac{\partial E_z}{\partial z} = 0 \tag{23}$$

For a lower frequency of a few MHz, laminated CFRP composites can be assumed to be homogeneous conductive materials [9]. This indicates that the dielectric constant is isotropic and, thus, the dielectric constant ε_0 can be used here:

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0 \tag{24}$$

Substitution of Equation (24) in Equation (22) gives:

$$\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} = \mu_0\sigma_x\frac{\partial E_x}{\partial t} \tag{25}$$

Similarly, the following equations can be derived:

$$\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial y^2} + \frac{\partial^2 E_y}{\partial z^2} = \mu_0\sigma_y\frac{\partial E_y}{\partial t} \tag{26}$$

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2} = \mu_0\sigma_z\frac{\partial E_z}{\partial t} \tag{27}$$

Thus, Equations (25–27) are Maxwell's equations for orthotropic CFRP composites when electric current flows in the x or y direction.

For cases where the electric current flows in the direction of an off-axis coordinate to the fiber direction, the conductance of the ξ-η coordinate that inclines by rotation angle θ is derived as follows [5]:

$$\begin{aligned} \sigma_{\xi\xi} &= \sigma_x \cos^2\theta + \sigma_y \sin^2\theta \\ \sigma_{\xi\eta} &= \sigma_{\eta\xi} = -(\sigma_x - \sigma_y)\sin\theta\cos\theta \\ \sigma_{\eta\eta} &= \sigma_y \cos^2\theta + \sigma_x \sin^2\theta \end{aligned} \tag{28}$$

Equations (25–27) for the rotated ξ-η coordinate then become as follows:

$$\frac{\partial^2 E_\xi}{\partial \xi^2} + \frac{\partial^2 E_\xi}{\partial \eta^2} + \frac{\partial^2 E_\xi}{\partial z^2} = \mu_0 \sigma_{\xi\xi} \frac{\partial E_\xi}{\partial t} + \mu_0 \sigma_{\xi\eta} \frac{\partial E_\eta}{\partial t} \tag{29}$$

$$\frac{\partial^2 E_\eta}{\partial \xi^2} + \frac{\partial^2 E_\eta}{\partial \eta^2} + \frac{\partial^2 E_\eta}{\partial z^2} = \mu_0 \sigma_{\eta\eta} \frac{\partial E_\eta}{\partial t} + \mu_0 \sigma_{\xi\eta} \frac{\partial E_\xi}{\partial t} \tag{30}$$

$$\frac{\partial^2 E_z}{\partial \xi^2} + \frac{\partial^2 E_z}{\partial \eta^2} + \frac{\partial^2 E_z}{\partial z^2} = \mu_0 \sigma_z \frac{\partial E_z}{\partial t} \tag{31}$$

3. Skin effect of electric current in unidirectional CFRP

To simplify the problem, let us consider a case where the electric field is polarized in the x-direction (fiber direction) and the electric field propagates in the z-direction, as shown in Figure 1. As the E_y and E_z electric fields do not exist in this case, Equation (25) is the only active equation. As the polarized E_x has uniform distribution in the x-direction and y-coordinate, Equation (25) can be rewritten as given in Equation (32):

$$\frac{\partial^2 E_x}{\partial z^2} = \mu_0 \sigma_x \frac{\partial E_x}{\partial t} \tag{32}$$

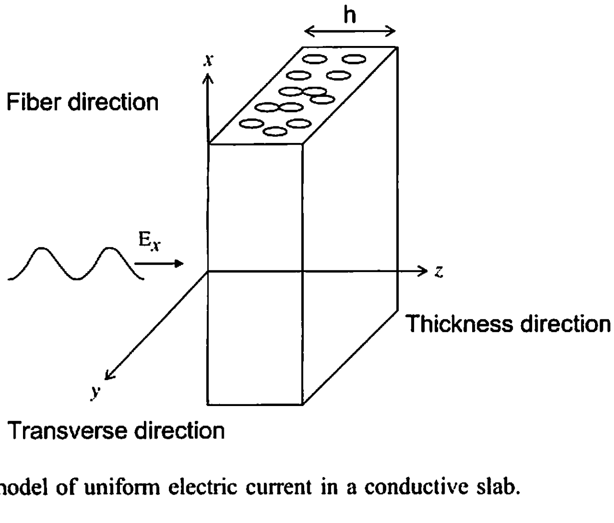

Figure 1. Simple model of uniform electric current in a conductive slab.

Let us consider the case that $E_x = E_0 e^{-j\omega t}$ is applied at the surface $z = 0$. The solution of Equation (32) is given as follows:

$$E_x = E_0 e^{-j\omega t} e^{-kz} \tag{33}$$

where, j is an imaginary number and k is an unknown complex number coefficient. Substation of Equation (33) in Equation (32) gives:

$$k = \pm(1 + j)\sqrt{\frac{\mu_0 \omega \sigma_x}{2}} \tag{34}$$

Let us focus on the positive time region of Equation (34),

$$k = (1 + j)\sqrt{\frac{\mu_0 \varpi \sigma_x}{2}} \tag{35}$$

$$\begin{aligned} E_x &= E_0 e^{-j\omega t} e^{j\sqrt{\frac{\mu_0 \sigma_x \omega}{2}} z} e^{-\sqrt{\frac{\mu_0 \sigma_x \omega}{2}} z} \\ &= E_0 e^{-j\omega\left(t - \sqrt{\frac{\mu_0 \sigma_x}{2\omega}} z\right)} e^{-\sqrt{\frac{\mu_0 \sigma_x \omega}{2}} z} \\ &= E_0 e^{-j\omega(t - \alpha z)} e^{-\beta z} \end{aligned} \tag{36}$$

where,

$$\alpha = \sqrt{\frac{\mu_0 \sigma_x}{2\omega}}, \quad \beta = \sqrt{\frac{\mu_0 \sigma_x \omega}{2}} \tag{37}$$

The term αz in Equation (36) indicates phase shift of the electric field caused by the distance (z) from the surface ($z = 0$). The term βz indicates attenuation of the amplitude of E_x caused by the distance (z) from the surface. Skin effect depth is defined as the distance where the amplitude of E_x becomes 1/e of the surface E_x. This skin effect depth δ_a can be obtained by solving the equation $\beta z = 1$ with respect to z.

$$\delta_a = \sqrt{\frac{2}{\mu_0 \sigma_x \omega}} \tag{38}$$

Replacement of σ_x in Equation (38) with the normal conductance of a metallic material yields the simple skin effect depth of the metallic material. For orthotropic CFRP composites, the skin effect depth is simply obtained by replacement of the electric conductance with the orthotropic conductance at which the electric current flows.

When the electric current flows in an off-axis angle, the equation includes a coupling term between the electric field E_ξ and E_η. When this coupling term exists, it is quite difficult to solve the equations and, therefore, this is not dealt with here.

4. Skin effect depth of cross-ply laminates

Let us consider the case of cross-ply laminates where fiber angles are limited to 0°/90° or ±45°. Let us assume that electric current flows in the fiber direction of the surface ply or the transverse direction (see Figure 2). The thickness direction is the z-coordinate, and the thickness of the laminates is assumed to be infinite. Here, let the thickness of each ply be h_1, h_2,

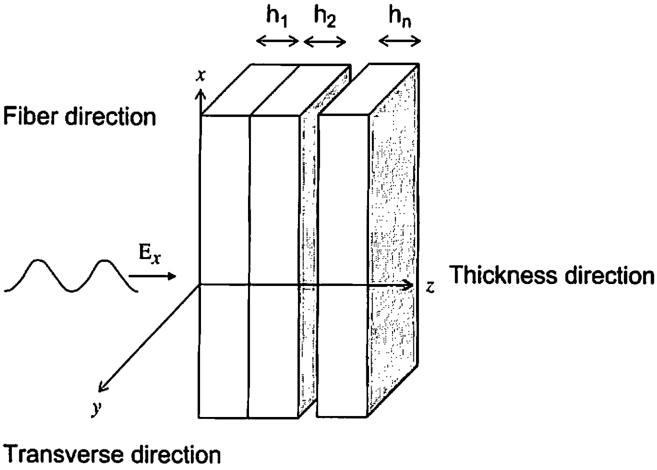

Figure 2. Simple model of uniform electric current in a laminated conductive slab.

..., h_n,... The corresponding electric conductance of each ply in the x-direction is $\sigma_{x1}, \sigma_{x2}, .., \sigma_{xn}, \ldots$ The electric conductance of each ply is not always in the fiber direction and is instead in the transverse direction for some plies. Similar to the case of a unidirectional CFRP, let us consider the case that the electric field is polarized in the x-direction and the electric field propagates in the z-direction. The electric field of the first ply is the same as that of Equation (36); the difference is that the conductance is σ_{x1}. α and β then become as follows:

$$\alpha_1 = \sqrt{\frac{\mu_0 \sigma_{x1}}{2\omega}}, \quad \beta_1 = \sqrt{\frac{\mu_0 \sigma_{x1} \omega}{2}} \tag{39}$$

As the electric field of the x-direction has continuity even at the interface between plies, the solution of Equation (36), substituting $z = h_1$, gives the input electrical field of the second ply:

$$E_x = E_0 e^{-j\omega(t - \alpha_1 h_1)} e^{-\beta_1 h_1} \tag{40}$$

In Equation (39), time, amplitude, and z-coordinate are replaced as follows:

$$\begin{aligned} t_1 &= t - \alpha_1 h_1 \\ E_1 &= E_0 \mathrm{e}^{-\beta_1 h_1} \\ z_1 &= z - h_1 \end{aligned} \tag{41}$$

As the variable transformations of Equation (41) do not affect the partial differential equations in (32), the solution can be similarly derived as follows:

$$E_x = E_1 e^{-j\omega(t_1 - \alpha_2 z_1)} e^{-\beta_2 z_1} \tag{42}$$

where,

$$\alpha_2 = \sqrt{\frac{\mu_0 \sigma_{x2}}{2\omega}}, \quad \beta_2 = \sqrt{\frac{\mu_0 \sigma_{x2} \omega}{2}} \tag{43}$$

Similar to the second ply, the output of the $(n-1)$th ply (input of the nth ply) is derived as follows:

$$\begin{aligned} t_n &= t - \alpha_{n-1} h_{n-1} \\ E_n &= E_{n-1} e^{-\beta_{n-1} h_{n-1}} \\ z_n &= z_{n-1} - h_{n-1} \end{aligned} \tag{44}$$

The solution of the input value of Equation (44) of the nth ply is then obtained as follows:

$$E_x = E_n e^{-j\omega(t_n - \alpha_n z_n)} e^{-\beta_n z_n} \tag{45}$$

where,

$$\alpha_n = \sqrt{\frac{\mu_0 \sigma_{xn}}{2\omega}}, \quad \beta_n = \sqrt{\frac{\mu_0 \sigma_{xn} \omega}{2}} \tag{46}$$

$$E_n = E_0 \mathrm{e}^{-\beta_1 h_1} \mathrm{e}^{-\beta_2 h_2} \cdots \mathrm{e}^{-\beta_{n-1} h_{n-1}} \tag{47}$$

Let us consider the case that the E_0 attenuates to 1/e at the nth ply. The skin effect depth δ_n can be calculated as follows:

$$\sum_{m=1}^{n-1} \beta_m h_m + \beta_n \left(\delta_n - \sum_{m=1}^{n-1} h_m\right) = 1 \tag{48}$$

When each ply is sufficiently thin, the actual skin effect depth δ_n of the cross-ply laminate is derived by obtaining the number of the nth-ply:

$$\begin{aligned} 1 \geq \sum_{m=1}^{n} \beta_m h_m &= \sqrt{\tfrac{\mu_0 \omega}{2}} \sum_{m=1}^{n} \sqrt{\sigma_{xm}} h_m \\ \delta_n &\approx \sum_{m=1}^{n} h_m \end{aligned} \tag{49}$$

5. Skin effect depth of general laminates

When the fiber directions in a CFRP laminate are not limited to cross-ply laminates, it is quite difficult to obtain an exact analytical solution of Equations (29) and (30) because of the presence of the angled plies. It is, however, possible to obtain an approximated solution when each ply is sufficiently thin, because the coupling of adjacent angled plies can be assumed to produce an isotropic ply of double thickness that has uniform electric field (see Figure 3). This coupling of $\pm\theta$ angled plies enables us to obtain an approximation of the skin effect depth.

For example, let us consider the case of a laminate with [45/0/–45/90/.....]s. Coupling of the outermost 45°-ply with the third 45°-ply creates a double-thickness ± 45°-ply that has uniform isotropic conductance and a uniform electric field. The second 90°-ply and the fourth 0°-ply are also coupled and create a double-thickness 0/90°-ply that also has uniform isotropic conductance and a uniform electric field. As the electric field is continuous, the difference between the electric fields of the adjacent plies is small when the plies are thin enough. This

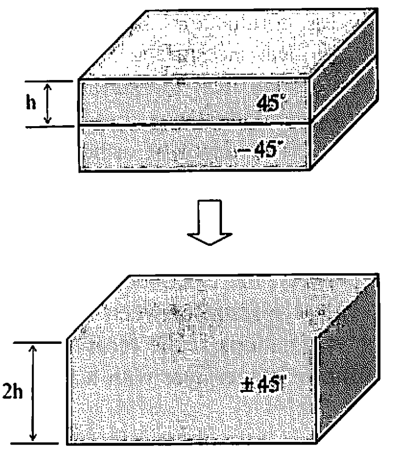

Figure 3. A coupled unit ply comprising two off-axis plies.

means that the electric fields of the adjacent plies have approximately similar values. The electric conductance of the coupled ±45°-ply of double thickness can be calculated using Equation (29) as follows:

$$\begin{aligned}\sigma_{\xi\xi} &= \sigma_{\eta\eta} = \tfrac{\sigma_x+\sigma_y}{2}\\ \sigma_{\xi\eta} &= \sigma_{\eta\xi} = 0\end{aligned} \tag{50}$$

The electric conductance in the fiber direction σ_x is, in fact, very much larger than that of the transverse direction σ_y [1]. This allows a simplified description of the double-thickness isotropic ply as follows:

$$\begin{aligned}\sigma_{\xi\xi} &= \sigma_{\eta\eta} \approx \tfrac{\sigma_x}{2}\\ \sigma_{\xi\eta} &= \sigma_{\eta\xi} = 0\end{aligned} \tag{51}$$

Similar results are obtained for the case of the coupling of 0/90°-plies. Using this approximation, the skin effect depth of the CFRP laminate can be obtained as follows:

$$\delta_a \approx 2\sqrt{\frac{1}{\mu_0\sigma_x\omega}} \tag{52}$$

This means that estimation of skin effect depth is √2≈1.41 times larger than that calculated from the electric conductance in the fiber direction, and that the skin effect depth is independent of the electric conductance in the thickness direction.

6. Comparison of the skin effect with the effect of anisotropic conductance

The skin effect depth of a direct current is easily calculated by substitution of 0 into ω for the solution of δ_a yielding an infinite value of δ_a. The solution of the skin effect depth is based on a uniform electric field for the z-coordinate, so in the case of a direct electric current, the current flow is uniform for uniform electric field E_x. For an alternating current, however, the induced electric current reduces the electric current even for uniform electric fields, the skin effect.

For CFRP laminates, the electric conductance in the thickness direction σ_z is very small, and the electric current concentrates near the surface even far from the electric current source [4]. This concentration of electric current near the surface of a CFRP laminate is defined as the skin effect caused by orthotropic conductance. Let us compare the skin effect depth of alternating current δ_a and the skin effect depth of orthotropic conductance δ_d when electric current is applied to the surface of a thick 0°-ply in the fiber direction. The skin effect depth of orthotropic conductance is defined as the distance from the surface at which the electric current density becomes 1/e of the surface current, at the middle point between the two electrodes. Three types of spacing between the electrodes are investigated here: 0.5, 1, and 2 m. These spacings correspond to typical spacing between metallic parts of actual aircraft structures. The material used for the calculations is the recently adopted highly toughened CFRP IM600/133. This CFRP has a resin-rich interlayer with a strongly orthotropic conductance, as shown in Table 1.

Electric current density i_x between two direct current electrodes in unidirectional CFRP can be easily obtained from the perfect fluid potential flow analysis [4]. In this analysis, the spacing is $2a$ and the applied electric current is I:

$$i_x = \frac{I}{\pi\sqrt{\sigma_x\sigma_z}}\left\{\frac{x+a}{\frac{(x+a)^2}{\sigma_x}+\frac{z^2}{\sigma_z}} - \frac{x-a}{\frac{(x-a)^2}{\sigma_x}+\frac{z^2}{\sigma_z}}\right\} \tag{53}$$

Because the longest distance point from the electrodes is the middle point between electrodes ($x=0$), the electric current density of the middle point is obtained as follows:

$$i_x = \frac{2aI\sqrt{\sigma_x\sigma_z}}{\pi(\sigma_z a^2 + \sigma_x z^2)} \tag{54}$$

The fraction Δ of electric current density at distance z from the surface against the electric current density of the surface ($z=0$) is given as follows:

$$\Delta = \frac{\frac{2aI\sqrt{\sigma_x\sigma_z}}{\pi(\sigma_z a^2+\sigma_x z^2)}}{\frac{2aI\sqrt{\sigma_x\sigma_z}}{\pi\sigma_z a^2}} = \frac{\sigma_z a^2}{\sigma_z a^2 + \sigma_x z^2} \tag{55}$$

When Δ is equal to 1/e, the distance z from the surface is the skin effect depth of orthotropic conductance δ_d. The δ_d is given as follows:

$$\delta_d = \sqrt{e-1}\sqrt{\frac{\sigma_z}{\sigma_x}}a \approx 1.311\sqrt{\frac{\sigma_z}{\sigma_x}}a \tag{56}$$

Equation (56) for IM600/133 is then as follows:

$$\delta_d = 0.3a \times 10^{-3}\ [m] \tag{57}$$

Table 1. Electric conductance of IM600/133.

σ_x (fiber) (S/m)	σ_y (transverse) (S/m)	σ_z (thickness) (S/m)
36,000	1.15	0.0018

From Equation (57), when the spacing is $2a=0.5$, 1, or 2 m, $\delta_d=0.075$, 0.15, or 0.3 mm, respectively.

The skin effect depth of alternating current δ_a for unidirectional CFRP, calculated using Equation (38), is shown in Figure 4. The abscissa is the logarithm of frequency, and the ordinate is δ_d. From Figure 4, even when the frequency of the applied alternating current is 1 MHz, the skin effect depth of alternating current δ_a is 2.65 mm.

Metallic fasteners are located approximately within 2 m of each other for typical aircraft. Thus, letting the spacing between electrodes be 2 m, the resulting skin effect depth of orthotropic conductance is $\delta_d=0.3$ mm. The frequency giving the same value for alternating current skin effect depth is 80 MHz.

When the frequency is 500 kHz, the alternating current skin effect depth is δ_a is approximately 3.8 mm. The alternating current of a lightning strike, which has frequency from μsec to msec, has a smaller skin effect depth of anisotropic conductance than of alternating current. This means the effect of orthotropic conductance is more significant than the skin effect of alternating current.

As shown by the lamination theory of electric conductance [5], there are two calculation methods for electric conductance of CFRP laminates. The first is a thin FRP laminate theory where the laminate is sufficiently thin and the electric field is uniform in the thickness direction. The second is a thick CFRP lamination theory where the electric field is not uniform and is obtained using direct current analysis.

As shown in the comparison of the skin effect depth, the orthotropic conductance greatly affects the electric conductance of CFRP laminates. Using the thin CFRP lamination theory, we can calculate the electric conductance of the CFRP laminate, and electric current can be calculated using the quasi-steady flow analysis derived from Maxwell's equations with the obtained electric conductance. However, this gives a large calculation error because the actual electric field is not uniform in the thickness direction and the thin CFRP lamination theory is not appropriate for nonthin CFRP laminate. For IM600/133, the effect of the alternating current is not larger than the effect of orthotropic conductance. Using the direct current analysis derived from the potential flow [4,5] with consideration of the effect of the orthotropic conductance gives more appropriate results.

As another case, let us consider a general-purpose CFRP cured at 130 °C [10]; this has a weaker orthotropic conductance than IM600/133. Equation (57) then becomes as follows:

$$\delta_d \approx 1.311\sqrt{\frac{3.3}{4100}}\,a = 0.0372a\,[m] \tag{58}$$

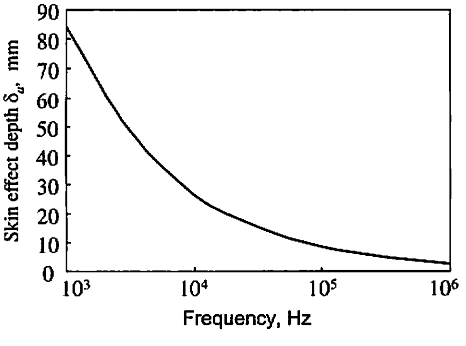

Figure 4. Skin-effect depth of IM600/133 in the fiber direction.

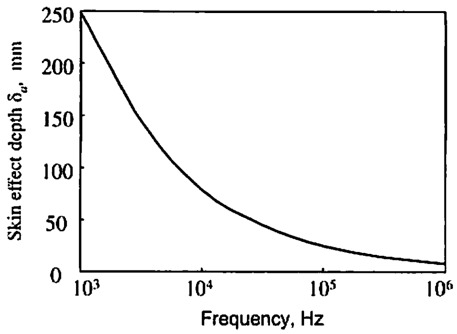

Figure 5. Skin effect depth in the fiber direction of a widely used CFRP.

When the spacing between electrodes is $2a = 2$ m, the skin effect depth of orthotropic conductance is $\delta_d = 37.2$ mm. When the frequency is 500 kHz, the skin effect depth of alternating current is $\delta_a = 11.1$ mm. The skin effect depth of the alternating current δ_a is smaller than the skin effect depth of orthotropic conductance δ_d for this CFRP. This means that the skin effect of alternating current is important in this case. Figure 5 shows the results of the skin effect depth of alternating current for this general-purpose CFRP.

When the spacing between the electrodes is $2a = 50$ cm, the skin effect depth of orthotropic conductance is $\delta_d = 9.3$ mm. In this case, the skin effect depth of orthotropic conductance is smaller than that of the 500 kHz alternating current. This indicates that direct-current analysis using potential flow of perfect fluid is sufficient for general-purpose CFRP when spacing between the electrodes is smaller than 50 cm. When the spacing is larger than 50 cm, quasi-steady state flow analysis by solving Maxwell's equations for the electromagnetic field induced electric current is required, because of the difference of the orthotropic conductance.

7. Conclusions

In the present study, quasi-steady state analysis of electromagnetic field was conducted to obtain the skin effect depth of alternating currents. The skin effect depth of alternating current was compared with the skin effect depth of orthotropic conductance. The results obtained are as follows:

(1) Skin effect depth of alternating current was obtained for a unidirectional CFRP. The skin effect depth equation simply requires substitution of the electric conductance in the fiber direction or transverse direction.
(2) Skin effect depth of alternating current was obtained for cross-ply CFRP laminates.
(3) Skin effect depth of alternating current was approximately obtained for general CFRP laminates.
(4) For highly toughened CFRP having strongly orthotropic conductance, the thick CFRP lamination theory of direct current was found to be effective up to a frequency of several MHz without requiring consideration of quasi-steady state electromagnetic field analysis.
(5) For a general CFRP laminate having weaker orthotropic conductance than the toughened CFRP, the thick CFRP lamination theory of direct current is only effective when the spacing between electrodes is short.

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