Unsupervised Structural Damage Diagnosis Based on Change of Response Surface Using Statistical Tool*
(Application to Damage Detection of Composite Structure)

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Most structural health monitoring systems adopt parametric methods based on modeling or non-parametric methods such as artificial neural networks. The former methods require modeling of each structure, and the latter methods require a large number of data for training. These methods demand high costs, and it is impossible to obtain training data of the damaged state of an in-service structure. By the present method, damage is detected by judging the statistical difference between data of the intact state and the current state. The method requires data of the undamaged state, but does not require complicated modeling or data for training. As an example, the present study deals with the detection of delamination of a composite beam. Damage is detected from the change of strain data using statistical tools such as the response surface and F-statistics. As a result, the new method successfully diagnoses the damage without the need to use modeling or data of the damaged state.

Key Words: Smart Structure, Damage Diagnosis, Response Surface, Statistical Tool

1. Introduction

Structural health monitoring systems are equipped with many sensors and are used to evaluate the damage state of an entire structure or structural components in real time. In order to prevent serious failure of civil structures such as bridges and gas pipes, the structural health monitoring system has recently become a technology of interest[1]-[4]. Particularly in Japan, it is necessary to minimize seismic disaster, and the development of a system for diagnosing the condition of civil structures in a short time at low cost is an urgent subject. The present research proposes a new damage diagnostic method for structural health monitoring of in-service structures, and the method is investigated experimentally.

Many damage diagnosis methods for structures have been proposed. Most of them employ either a parametric method based on modeling of the structure or a nonparametric method such as the artificial neural network (ANN). As parametric methods, a substructural flexibility method[5],[6] and an electrical resistance change method[7] have been proposed. As nonparametric methods, ANNs[8],[9] and the response surface method (RSM)[10],[11] have been proposed. In both types of methods, damage is diagnosed by identifying the relationship between damage and sensor output and damage is identified from actual sensor measurement. Therefore, damage can be identified in detail from sensor data. However, in general, these

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methods require modeling of each structure or a vast amount of learning data to identify the relationship between the sensor output and damage condition of the structure. Moreover, destructive experiments on existing structures are unfeasible, it is impossible to obtain data for training on the damaged state of structures in the case of a structural health monitoring system of existing structures. This is the reason behind the importance of developing a method for diagnosing damage using only undamaged state data of the structure.

In the method, structural damage is diagnosed by using statistical similarity tests of the response surface, which show the relationships between the output of sensors. The method requires only data sets for the nondamaged state, and does not require complicated modeling or numerous data sets after the generation of damage for training ANNs, thereby considerably lowering the cost of creating a diagnostic system.

In the present study, as an example of damage diagnosis by this method, the method is applied to delamination detection of a CFRP beam, and the new damage diagnosis method is investigated experimentally.

2. Damage Diagnosis Method Using Statistical Similarity Test of Response Surfaces

In this section, the new damage detection method using statistical tools is addressed. Since damage of a structure causes a change in the output of a sensor attached to the structure, damage can be detected if boundary conditions are strictly determined and no noise is included in sensor measurement. However, in general, it is impossible to apply the same boundary conditions strictly, and damage diagnosis only by comparing the absolute value of sensors is nearly impossible. For conventional health monitoring systems, the relationships between the measured sensor data and damage location or size are indispensable for identifying damage. These relationships are derived by modeling the entire structure or through experiments. Such techniques for obtaining the relationships are very time-consuming and require high computational and/or experimental cost.

The new statistical diagnostic method proposed in the present paper is a simple low-cost system. The diagnostic method determines the relationships between sensors using the response surface, and the damage is automatically diagnosed by detecting the change of this response surface using statistical tests.

2.1 System identification using response surface methodology

In this method, response surface methodology is use to elucidate the relationship between sensor outputs. Response surface methodology is employed for process optimization in a quality-engineering field. Response surface methodology consists of a design of experiments to select the most suitable points for fitting the surfaces effectively, the least squares method to regress response surfaces and optimization of the regression model using the t-test. The response surface is the approximation function that expresses the relationship between a response and predictors. Generally, a response surface is represented by

$$y=f(x_1, x_2, \cdots, x_l)+\varepsilon, \qquad (1)$$

where x are predictors, y is a response, ε is regression error and l is the number of predictors. In general, polynomials are used for the response surface.

Advantages of solving the relationship between sensor outputs using the response surface are as follows.

1. Since the significance of each regression coefficient is derived from the statistical t-test, the optimal regression model for the response surface can be selected automatically.
2. Number of experiment need for response surface of high accuracy are reduced by using experimental design method.

2.2 Least-square-method

For simplification, let us consider the case in which a response is approximated by quadratic polynomials of predictors as follows.

$$y=\beta_0+\sum_{j=1}^{l}\beta_j x_j+\sum_{j=1}^{l}\beta_{jj}x_j^2+\sum_{i=1}^{l-1}\sum_{j=i+1}^{l}\beta_{ij}x_i x_j \qquad (2)$$

β is the regression coefficient. If squares or interactions of the predictors x_j^2 and $x_i x_j$ are replaced by new predictors x_j $(j>l)$, formula (2) becomes the following linear regression model:

$$y=\beta_0+\sum_{j=1}^{k}\beta_j x_j \qquad (3)$$

where k is the number of predictors after the replacement. In terms of n observations, Eq.(3) can be written in matrix form

$$\boldsymbol{Y}=\boldsymbol{X\beta}+\boldsymbol{\varepsilon}, \qquad (4)$$

where $\boldsymbol{Y}$, $\boldsymbol{\beta}$, $\boldsymbol{\varepsilon}$ and $\boldsymbol{X}$ are matrix forms of response, regression coefficient, error and predictor variable.

$$\boldsymbol{Y}=\begin{Bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{Bmatrix},\ \boldsymbol{\beta}=\begin{Bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_k \end{Bmatrix},\ \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{Bmatrix} \qquad (5)$$

$$\boldsymbol{X}=\begin{bmatrix} 1 & x_{11} & x_{12} & \cdots & x_{1k} \\ 1 & x_{21} & x_{22} & \cdots & x_{2k} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_{n1} & x_{n2} & \cdots & x_{nk} \end{bmatrix} \qquad (6)$$

An unbiased estimator of $\boldsymbol{\beta}(\boldsymbol{b})$ is obtained using the least-squares method as follows:

$$\boldsymbol{b}=(\boldsymbol{X}^T\boldsymbol{X})^{-1}\boldsymbol{X}^T\boldsymbol{Y}. \qquad (7)$$

2.3 Significance test of predictors

To derive the optimum regression model for the response surface, the statistical t-test has been performed by the following procedure.

1. Determine the regression coefficient using the least-squares method.
2. Test the significance of each term of the regression model using the t-test.
3. Delete the term which has the worst t-statistical value when the term derives worth regression result.
4. Repeat step 1 to 3, when the term which yields the derive worth regression result exists.

The significance of each term of the response surface is tested by the statistical t-test. In order to investigate the significance of a term, a null hypothesis is introduced. The hypothetical definition is

$$\beta_i=0, \tag{8}$$

where β and i are the regression coefficient and term tag shown in formula (3). Let us assume that regression error follows a normal distribution $N(0, \sigma^2)$. In this case, the t-statistic value of term $i(t_i)$ is defined as (10)

$$t_i=\frac{b_i}{\sqrt{\dfrac{\sum_{j=1}^{n}(y_j-\hat{y}_j)^2/(n-k-1)}{\sum_{j=1}^{n}(x_{ij}-\bar{x}_i)}}}, \tag{9}$$

where b_i is the least-squares estimator of β. Under the null hypothesis, this t-statistic value t_i follows the t-distribution of degrees of freedom n. The critical region of the hypothesis is represented by the following formula.

$$t_i > t_{\alpha/2,n-k-1} \tag{10}$$

The significance of term i is rejected when t_i is smaller than $t_{\alpha/2,n-k-1}$. The term which has the smaller t_i is deleted to form a good regression model. This process is repeated to derive the best regression model for the response surface.

2.4 Similarity test of response surfaces using F-test

A similarity test of two response surfaces is performed by the statistical F-test, which is generally used for testing the similarity of two distributions.

2.4.1 Calculation of F-test value F_0 of similarity test We have two response surfaces that are created from two different experiments:

$$Y_1=X_1\beta_1+\varepsilon_1$$
$$Y_2=X_2\beta_2+\varepsilon_2' \tag{11}$$

where the number of experiments for regression are n_1 and n_2, respectively. In order to investigate the similarity between the two response surfaces, a null hypothesis is introduced. The hypothetical definition is shown as

$$H_0:\ \beta_1=\beta_2. \tag{12}$$

An alternative hypothesis is shown as

$$H_1:\ \beta_1\neq\beta_2. \tag{13}$$

Assume that each error term (ε) is independent and has the same distribution in both experiments. In this case, the F-statistic value F_0 is defined as follows.

$$F_0=\frac{SSE_0-SSE_{12}}{SSE_{12}} * \frac{n-2p}{p} \tag{14}$$

$$n=n_1+n_2 \qquad SSE_{12}=SSE_1+SSE_2$$

Here, SSE with subscript 0 means SSE of the response surface with all data. SSE represents the residual sum of squares of a response surface as follows.

$$SSE_i=Y_i^TY_i-b_i^TX_i^TY_i \tag{15}$$

The F-statistic value F_0 follows a F-distribution of degrees of freedom (p, $n-2p$) under the null hypothesis H_0. When the two response surfaces are similar to each other, F_0 becomes a small value.

2.4.2 Critical value of hypothesis H_0 In general, the probability distribution of the F_0 value of the F-similarity test for similar multiple regression models follows the $F(p, n-2p)$ distribution. For this reason, critical value of hypothesis H_0 is determined from the significance level α, p and n. The critical region hypothesis H_0 is shown as the following formula.

$$F_0 > F^{\alpha}_{p,n-2p} \tag{16}$$

The similarity of response surfaces is rejected when F_0 is larger than $F^{\alpha}_{p,n-2p}$. Thus, the critical region of the similarity test of response surfaces is determined only from the significant level and the model of the response surface.

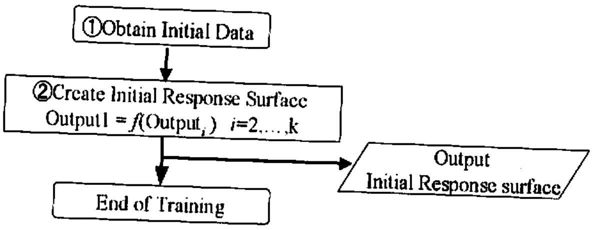
In the F similarity test, significant level α is called the Error of the First Kind (probability of incorrect rejection). Since the Error of the First Kind is due to the significant level α, the diagnostic accuracy of the F-test is evaluated from $1-\beta$, where β represents the probability of incorrect acceptance, which is called the Error of the Second Kind. This $1-\beta$ called as power. The accuracy of the test is verified using the power.

2.5 Damage diagnosis method using response surface and F-test

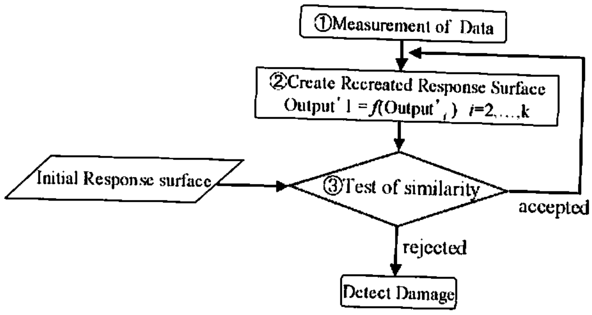
In this method, damage is diagnosed by detecting the change of the response surface due to damage. Where response surface shows relation between outputs of each sensor. The damage detection procedure is as follows.

(1) Training mode

The training process is required before starting the monitoring. During this process, the response surface of intact state is defined. The learning process is shown in Fig. 1(a). First, a response surface is calculated from the sensor data measured from the intact state structure. This response surface is named as initial response surface. Data from a sensor is



(a) Flow of training



(b) Flow of monitoring

Fig. 1 Flow of damage detection

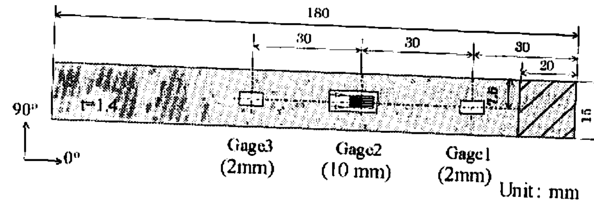


Fig. 2 Specimen configuration

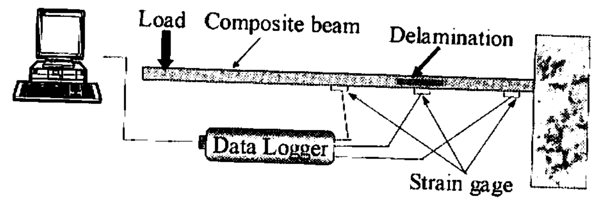


Fig. 3 Experimental setup

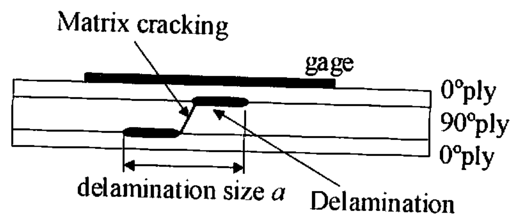


Fig. 4 Schema of delamination

selected as the response and the data obtained from the adjacent sensors are selected as predictors.

(2) Monitoring mode

Procedure of the monitoring process is shown in Fig. 1(b). During the monitoring process, a set of every sensor data is periodically obtained by repeating measurements several times. From the set of measured data, a response surface is recreated. This response surface is named as recreated response surface here. The two response surfaces are compared with each other using the statistical similarity test with F-test. When the recreated response surface is discriminated from the initial response surface, it means that the relationship between the sensor data has changed. Such a change means that something has happened in the structure, leading to the conclusion that the structure has been damaged.

3. Detection of Delamination Crack Using Change in Surface Strain Distribution

In the present study, the new statistical diagnostic method is applied to delamination detection in a CFRP beam, and the effectiveness of the method is experimentally investigated. The generation of a delamination crack is detected using the change in the surface strain distribution.

3.1 Specimen for experiment

The raw material used to fabricate the specimen is unidirectional prepreg TR340M150ST produced by Mitsubishi Rayon Co. Limited. Laminates with a staking sequence of $[0_2/90_2]_s$ were fabricated by a hot press process. The curing temperature was 130°C, the curing time was one hour and the pressure was 1.1 MPa. The beam specimens were cut out from the cured laminates. The dimensions are 180 mm length and 15 mm width, as shown in Fig. 2. The thickness of the specimen is about 1.4 mm.

Conventional strain gages are employed as sensors. Three strain gages are mounted on the specimen surface in the arrangement shown in Fig. 2. The center gage has a gage length of 10 mm, and the side gages have a gage length of 2 mm. In the experiments, the right end of the specimen is held the left end of the specimen is excited. Time series data are measured under the excitation condition. A delamination crack is created under the center gage by means of a short beam shear test. Figure 4 shows a schematic of the delamination. Delamination occurs between 0° and 90° plies, initiated by matrix cracking in the middle 90° ply. The length of delamination (a) is defined as illustrated in Fig. 4.

3.2 Response surface

In the present study, strain data from center gage #2 are considered to be the response, and the strain data from gages #1 and #3 are considered to be the predictors. The response surface is created using quadratic polynomials and is approximated as

$$\varepsilon_2=\beta_0+\beta_1\varepsilon_1+\beta_2\varepsilon_3+\beta_3\varepsilon_1^2+\beta_4\varepsilon_1\varepsilon_3+\beta_5\varepsilon_3^2, \quad (17)$$

where ε_i represents the strain data measured from gage #i. As shown in formula (17), the degrees of freedom of the response surface is 6. The response surface of the equation is regressed from 15 strain

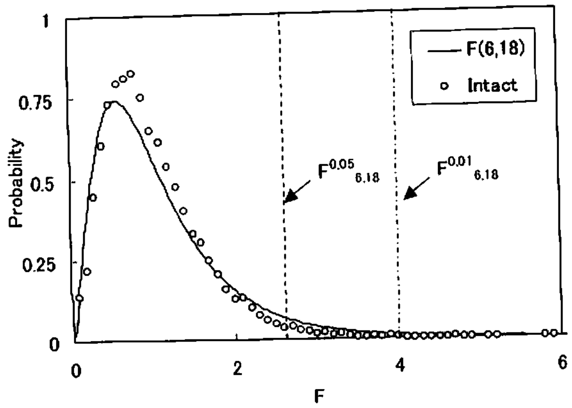


Fig. 5 F-distribution of intact specimen

data sets measured from the strain gages.

3.3 Critical region of similarity test

As shown in section 2.4, the probability distribution of the F_0 value in a similarity test of the similar response surfaces follows $F(p, n-2p)$. In this case, the degrees of freedom p is 6, total number of response surfaces is $n=15\times2=30$, and the probability distribution of F_0 in the intact specimen may follow $F(6, 18)$. Figure 5 shows experimental values of the probability distribution of F_0 of an intact specimen and the $F(6, 18)$ distribution. In the experiment, both the initial response surface and the recreated response surface are created from measured data taken from the same intact specimen. The number of trials is 2 000. As shown in Fig. 5, the experimental distribution and theoretical distribution are in good agreement.

Therefore, the probability distribution of the F_0 value of the intact structure may follow a distribution that depends on the model shape of response surfaces, and the critical value of the F-similarity test, which is the threshold value between the intact state and a damaged state, is derived from only a number of data sets and the degree of freedom of response surfaces. This result indicates that in the present method, the threshold value between the intact and damaged states is not affected by noise, variance, or average value of sensor measurement.

In this study, the significance level α for the F-similarity test is set to 1% and 5%. In this case, critical values are $F^{0.01}_{6,18}=4.02$ and $F^{0.05}_{6,18}=2.66$. When F_0 exceeds one of these values, similarity between the identified systems is rejected and the structure is diagnosed as being damaged.

3.4 Diagnostic accuracy of delamination crack detection

Using the critical values calculated at significant levels of 95% and 99%, performance tests of the diagnostic method are conducted to investigate its effectiveness. After creating a delamination crack, the recreated response surface is calculated from 15 strain data sets measured from same sensors.

Table 1 Region of delamination length and number of experiments

Delamination length[mm]	Number of repetitions of experiment
Intact	2000
2<a≦4	180
4<a≦6	180
6<a≦8	144
8<a≦10	108

Table 2 Average F_0 of each size category

Delamination length[mm]	Average of F_0
Intact	1.3
2<a≦4	11.0
4<a≦6	49.9
6<a≦8	84.5
8<a≦10	206.8

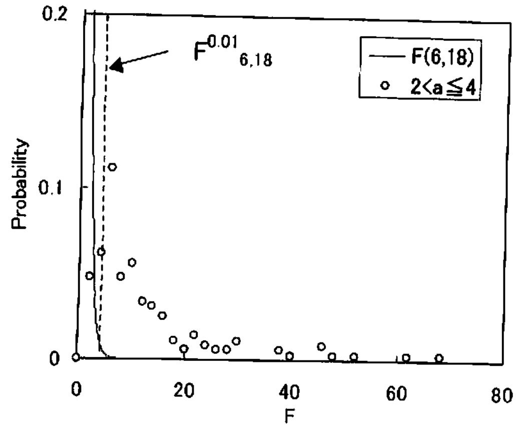
Due to the experimental difficulties of creating delamination cracks of exact lengths, delamination lengths are quantized into four categories in increments of 2 mm. Table 1 shows the number of experiments on each delamination length categories.

Figure 6 shows the plot of the probability distribution of the damaged specimen. The solid line in the figure shows the F-distribution under the intact condition and the dotted line shows the critical value at the significant level of 1%. As shown in the figures, the F-distribution of the damaged specimen is clearly changed from that of the intact specimen, and the f-value is higher than the critical value at high probability. Table 2 shows the average value of F_0 statistics for each size category. F_0 increases from 1.3 to 206.8 uniformly according to the increase of the delamination length. The diagnostic accuracy of the method increases with increasing amount of the damage.

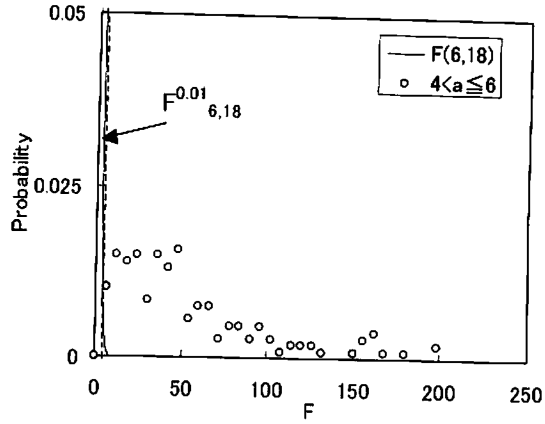
To consider the influence of the significant level, the diagnostic accuracy of each significant level in shown in Fig. 7. For the intact case (delamination size is zero), the similarity between the two response surfaces were significant at almost 99% & 95% accuracy. When the delamination crack length exceeds 6 mm, similarity between the two response surfaces is rejected at a rate of almost 100%, which is a perfect diagnosis of the presence of delamination at both significant levels. To reduce error of the first kind, the significant level of 1% should be use for delamination detection in the CFRP beam.

4. Conclusions

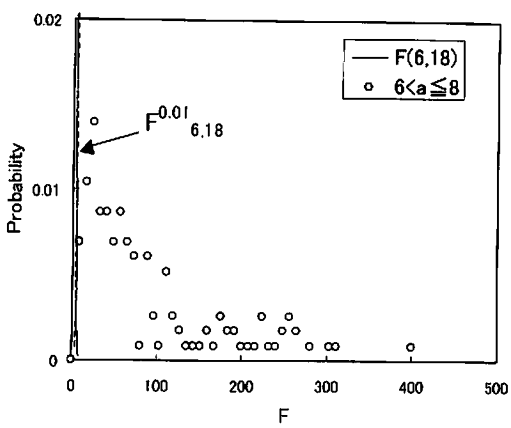
By conducting similarity tests on the two response surfaces which show relationships between output of each sensors, we propose a new damage diagnosis method that does not require consideration of the damaged state. As an example of damage



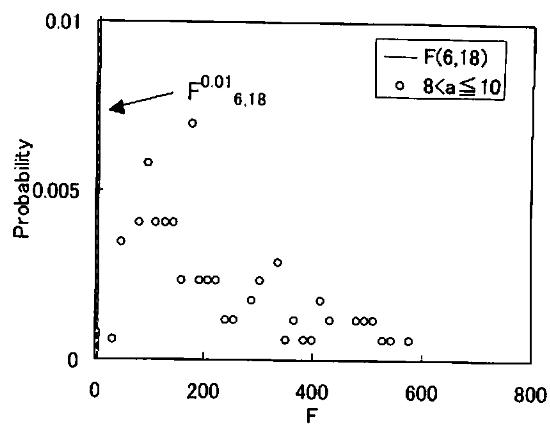
(a) $2<a\leqq4$



(b) $4<a\leqq6$



(c) $6<a\leqq8$



(d) $8<a\leqq10$

Fig. 6 F-distribution of damaged specimen

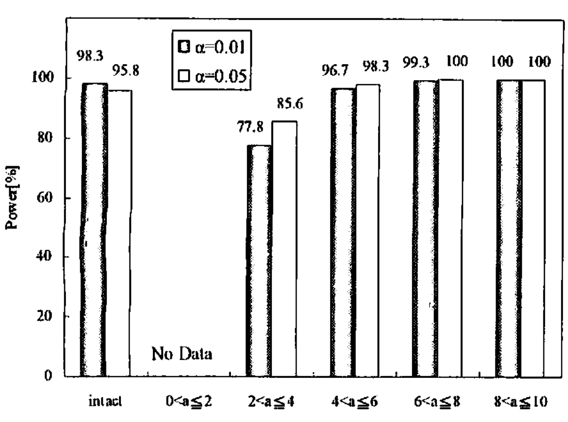


Fig. 7 Result of detection of delamination

diagnosis by this method, we describe delamination detection in a composite beam. Delamination was detected from slight changes in strain data measured by gages mounted on the specimen surface, using new statistical tools. The new method employs response surfaces and F-statistics to discriminate between two response surfaces obtained from different sets of measured data. The following results were obtained.

(1) By diagnosing the difference in sensor output with statistical tools such as response surface and F-distribution, damage can be detected merely by comparing sensor output data with initial data.

(2) A critical limit for discriminating the intact state from the damaged state can be defined from the F_0-distribution of the intact state. Therefore, the method does not require a large number of experiments on the damaged state.

(3) The F-similarity test at a significant level of 1% shows good accuracy of delamination detection in a CFRP beam.

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