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Stacking sequence optimizations using GA with zoomed response surface on lamination parameters

AKIRA TODOROKI* and MASAFUMI SASAI

Department of Mechanical Sciences and Engineering, Tokyo Institute of Technology, 2-12-1 Ohokayama, Meguro, Tokyo 152-8552, Japan

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Abstract—Stacking sequence optimizations for composite laminates are combinatorial optimization problems often handled by genetic algorithms (GA). Unfortunately, GAs often require large computational resources for individual evaluation. To overcome this problem, this paper proposes use of lamination parameters in an optimization process approach with GA. The optimization process requires precise approximation of design space using response surfaces to obtain the true optimal result. However, stacking sequence optimizations are discrete optimization problems and it is difficult to judge optimality of an obtained result on the lamination parameter space. For zooming response surfaces, we require an optimality check method. In this study, laminates adjacent to the obtained optimal result are introduced on the lamination parameter space and an optimality-check-test method is proposed. The test method is applied to a buckling load maximization problem of a laminated composite plate; its effectiveness is confirmed by example.

Keywords: Genetic algorithm; response surface; design of experiment; composites; optimization.

1. INTRODUCTION

Laminated composites are sought by the aerospace industry as attractive materials for aerospace structures because laminated composites have high specific strength and specific stiffness. Most laminated composites are fabricated from stacking unidirectional prepreg. Stacking sequence, which describes the order of fiber angles of the unidirectional prepreg, has a significant effect on laminated composite strength and stiffness; stacking sequence optimizations, therefore, are a significant problem for laminated composite design. One stacking sequence optimization method is a graphical design method with lamination parameters proposed by Miki [1] and Fukunaga and Chou [2], which employs lamination parameters as continuous design variables and obtains an optimal stacking sequence of an angle

*To whom correspondence should be addressed. E-mail: atodorok@ginza.mes.titech.ac.jp

ply laminate such as $\pm\theta$. For practical composite structures, however, available fiber angles are limited to a small set of 0°, ±45°, and 90° plies by lack of experimental data and its complicated fabrication process. These limitations give rise to stacking sequence optimizations which are combinatorial optimization problems with some constraints. Susuki [3] proposed a ranking method for strength optimizations of laminated composites for in-plane loadings. For general stacking sequence optimizations, genetic algorithms (GA) are becoming popular [4–13] because GAs are generally admitted to be effective for combinatorial optimizations. For stacking sequence optimizations, however, there are several combinatorial constraints which are normally difficult to implement in GAs. For example, to prevent a tension-shear coupling effect, the number of angle plies must be balanced. To prevent a large matrix cracking effect, stacking of more than four plies of identical fiber angles must be avoided. Penalty methods to implement these combinatorial constraints reduce GA performance; for this, a recessive-gene-like repair strategy has been proposed by some authors [14, 15].

The GAs require high computational cost because algorithms repeat evaluations for numerous genes. Yamazaki has proposed a two-stage method to reduce computational cost [7]: in the first stage, optimal lamination parameters are obtained with a mathematical programming method assuming lamination parameters are continuous independent variables; then, the most contiguous practical laminate is selected with a GA in the second stage. This method provides improved optimal results, but it does not always give a truly optimal result because in-plane lamination parameters and out-of-plane lamination parameters are not independent of each other for practical laminates. The authors proposed a new GA for stacking sequence optimizations using response surface approximations in lamination parameters [13]; that method excellently reduces computational cost. In that method, GA is employed to search an optimal stacking sequence because searching an optimal stacking sequence is a quite difficult problem with many local optimal points. Of course, in the case of thin laminates, it is easy to search all feasible laminates using response surface approximation of the objective function. For thick laminates, however, it is efficient to adopt a genetic algorithm to search optimal stacking sequence. Even for the genetic algorithm for stacking sequence optimizations, performance to obtain a true optimal laminate for thick laminates decreases with increased number of stacks. Some authors have proposed a new two-stage genetic algorithm using consanguineous initial population to obtain high performance [16].

The present study employs a GA proposed before [13]; it reduces computational cost and provides a simple interface between the GA and general analytical codes using response surfaces. Lamination parameters are adopted as variables of response surface approximations of the objective function. The GA performs optimizations using response surfaces for chromosome evaluation. Adoption of lamination parameters as variables yields the approximated objective function with a small number of calculations and reduces objective function non-linearity. Adoption of the response surface as an approximation method of the objective function also simplifies software development of the interface between the GA and a conventional calculation tool, such as FEM code.

In the method, the most important point is fitness of approximation with the response surface. When fitness of the response surface is unacceptable, a zoomed response surface must be employed as described in Ref. [13]. The previous paper, however, does not provide a criterion to decide response surface fitness. Generally, an optimality check test can give the criterion; such a check test uses contiguous points for normal integer programming. It is, however, very difficult to obtain adjacent laminates in lamination parameter coordinates. The present study, therefore, proposes a new criterion to decide response-surface approximation fitness for stacking sequence optimizations. The method is applied to a simple example to maximize buckling load and in-plane stiffness to check effectiveness, although the problem is not a practical one.

2. OPTIMIZATION PROBLEM

2.1. Buckling load and stiffness of laminate

As mentioned above, the optimization problem herein is not a practical problem. It merely confirms effectiveness of the method proposed here.

A simply supported symmetric rectangular plate considered here is shown in Fig. 1: plate length is a, plate width is b, and plate thickness is h. Reference loads λN_x, λN_y, λN_{xy} are applied; λ is a proportional loading coefficient. Consider that the laminate comprises a set of plies with fiber angles of 0°, ±45°, and 90° plies, and that the laminate is balanced symmetric. In this case, buckling load coefficient λ_n is calculated as

$$\lambda_n = \frac{\pi^2\{D_{11}(m/a)^4 + 2(D_{12} + D_{66})(m/a)^2(n/b)^2 + D_{22}(n/b)^4\}}{(m/a)^2 N_x + (n/b)^2 N_y}, \tag{1}$$

$$\lambda_s = \pm\frac{9\pi^4 ab}{32}\left[\frac{D_{11}}{a^4} + \frac{2(D_{12} + 2D_{66})}{a^2b^2} + \frac{D_{22}}{b^4}\right], \tag{2}$$

$$\begin{aligned} \lambda_{\text{cr}} &= \min\{|\lambda_s|, \lambda_c\}, \\ \frac{1}{\lambda_c} &= \frac{1}{\lambda_n} + \frac{1}{\lambda_s^2}, \end{aligned} \tag{3}$$

where m and n are half numbers of buckling waves in the x and y directions. Out-of-plane stiffness values of a symmetric laminate are obtained as

$$\begin{aligned} D_{11} &= \frac{h^3}{12}\left(U_1 + U_2 W_2^* + U_3 W_2^*\right), \\ D_{22} &= \frac{h^3}{12}\left(U_1 - U_2 W_1^* + U_3 W_2^*\right), \end{aligned} \tag{4}$$

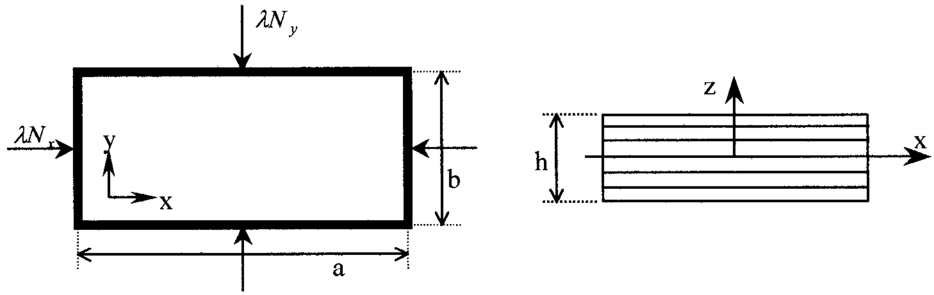


Figure 1. Configuration of laminated composite plate.

$$D_{12} = \frac{h^3}{12}\left(U_4 - U_3 W_2^*\right),$$
$$D_{66} = \frac{h^3}{12}\left(U_5 - U_3 W_2^*\right),$$

where U_i ($i = 1, \ldots, 5$) are material invariants and W_i^* ($i = 1, 2$) are out-of-plane lamination parameters described later. U_i ($i = 1, \ldots, 5$) are defined as

$$\begin{aligned}
U_1 &= \frac{1}{8}\left(3Q_{11} + 3Q_{22} + 2Q_{12} + 4Q_{66}\right),\\
U_2 &= \frac{1}{2}\left(Q_{11} - Q_{22}\right),\\
U_3 &= \frac{1}{8}\left(Q_{11} + Q_{22} - 2Q_{12} - 4Q_{66}\right),\\
U_4 &= \frac{1}{8}\left(Q_{11} + Q_{22} + 6Q_{12} - 4Q_{66}\right),\\
U_5 &= \frac{1}{8}\left(Q_{11} + Q_{22} - 2Q_{12} + 4Q_{66}\right),
\end{aligned} \tag{5}$$

where Q_{ij} are defined with orthotropic elastic modulus as follows:

$$\begin{aligned}
Q_{11} &= \frac{E_1}{1 - \nu_{12}\nu_{21}}, \quad Q_{22} = \frac{E_2}{1 - \nu_{12}\nu_{21}},\\
Q_{12} &= \frac{\nu_{12}E_2}{1 - \nu_{12}\nu_{21}} = \frac{\nu_{21}E_1}{1 - \nu_{12}\nu_{21}}, \quad Q_{66} = G_{12}.
\end{aligned} \tag{6}$$

A buckling load λ_c is calculated from m and n that minimize the value of λ_n. For a buckling load maximization problem, a stacking sequence to maximize a buckling load calculated from equation (3) must be obtained.

In-plane stiffness of a symmetric laminate in the x-direction can be obtained only with in-plane stiffness A_{11}, which is calculated as

$$A_{11} = h\left(U_1 + V_1^* U_2 + V_2^* U_3\right), \tag{7}$$

where V_i^* ($i = 1, 2$) are in-plane lamination parameters described later.

In the present study, the objective function f for maximization is defined as

$$f = \left(\lambda' \times 0.8 + \frac{A_{11}}{A'_{11}} \times 0.2\right) \Big/ (1 + p), \tag{8}$$

where λ' is the buckling load coefficient λ_c divided by $(N_s/2)^3$; since the buckling load is proportional to cubic of half number of stacks $(N_s/2)$, the buckling load is normalized by $(N_s/2)^3$ here. A'_{11} represents the maximum value of A_{11}, which is easily obtained from a laminate comprising only 0° plies. The value of p is a penalty for anisotropic laminates, which is 0.2 when anisotropic parameters δ and γ are larger than 0.2. Anisotropic parameters δ and γ are defined as follows [17]:

$$\gamma = \frac{D_{16}}{\left(D_{11}^3 D_{22}\right)^{1/4}}, \quad \delta = \frac{D_{26}}{\left(D_{22}^3 D_{11}\right)^{1/4}}. \tag{9}$$

Above, D_{16} and D_{26} are defined as the following:

$$D_{16} = \frac{h^3}{12}\left(U_2 W_3^*/2 + U_3 W_4^*\right), \quad D_{26} = \frac{h^3}{12}\left(U_2 W_3^*/2 - U_3 W_4^*\right). \tag{10}$$

Laminate size is $a = 0.508$ m, $b = 0.254$ m, and $h = 2$ mm here. The material is a typical graphite/epoxy composite T300/epoxy: $E_1 = 133$ GPa, $E_2 = 8.7$ GPa, $G_{12} = 3.2$ GPa, and $\nu_{12} = 0.26$. The total number of stacks N_s is 16 and available fiber angles are 0°, ±45°, and 90° plies. Stacking sequence optimizations in the present study, therefore, obtain a laminate of 16 plies that maximizes the value of equation (8) from the set of fiber angles of 0°, ±45°, and 90° plies. Weights of equation (8) of 0.8 and 0.2 are set to produce a complicated problem, and have no engineering meaning here. It is appropriate to obtain the objective function of complicated structures from FEM analyses, but the complicated objective function obtained from adding simple problems is adopted here to check true optimality.

The number of angle plies is balanced; the four-contiguous ply rule to avoid large matrix cracking is implemented with the recessive-gene-like repair strategy here [14, 15].

2.2. Lamination parameters

In-plane and out-of-plane lamination parameters of a symmetric laminate are given as

$$V_1^* = \frac{1}{h}\int_{-h/2}^{h/2} \cos 2\theta \, \mathrm{d}z,$$
$$V_2^* = \frac{1}{h}\int_{-h/2}^{h/2} \cos 4\theta \, \mathrm{d}z,$$
$$V_3^* = \frac{1}{h}\int_{-h/2}^{h/2} \sin 2\theta \, \mathrm{d}z, \text{ and} \tag{11}$$

$$
\begin{aligned}
V_4^* &= \frac{1}{h}\int_{-h/2}^{h/2} \sin 4\theta \, \mathrm{d}z, \\
W_1^* &= \frac{12}{h^3}\int_{-h/2}^{h/2} \cos 2\theta z^2 \, \mathrm{d}z, \\
W_2^* &= \frac{12}{h^3}\int_{-h/2}^{h/2} \cos 4\theta z^2 \, \mathrm{d}z, \\
W_3^* &= \frac{12}{h^3}\int_{-h/2}^{h/2} \sin 2\theta z^2 \, \mathrm{d}z, \\
W_4^* &= \frac{12}{h^3}\int_{-h/2}^{h/2} \sin 4\theta z^2 \, \mathrm{d}z.
\end{aligned} \tag{12}
$$

In those equations, z is the coordinate in the thickness direction whose origin locates at the middle of the laminate, θ is the fiber angle at location z, and h is laminate thickness.

All elements of stiffness matrix of laminates can be obtained from lamination parameters and material invariants mentioned before [18, 19]. Equations (11) and (12) show that the total number of lamination parameters is invariant and fixed at eight. In the present study, symmetric balanced laminates comprising the set of fiber angles of 0°, ±45°, and 90° plies are considered; the value of W_3^* almost vanishes because anisotropy constriction is implemented with anisotropic parameters. These conditions reduce the number of lamination parameters requiring consideration for approximations with response surfaces to four: V_1^*, V_2^*, W_1^*, and W_2^*. Adoption of lamination parameters as variables of approximation of the objective function brings significant advantages that avoid increase of variables for thick laminates and reduce design space non-linearity [20].

3. RESPONSE SURFACE METHODOLOGY [21]

Response surface methodology is applied to obtain approximation to a response function in terms of predictor variables. Methodology comprises experimental design, the least-square-error method, and optimizations.

In the present study, response y is obtained from equation (8) and variables are four lamination parameters: V_1^*, V_2^*, W_1^*, and W_2^*. Individual ply fiber angles are not adopted as variables; this prevents increase of variables for thick laminates and reduces non-linearity. The response surface is

$$
\begin{aligned}
y = {}& \beta_0 + \beta_1 V_1^* + \beta_2 V_2^* + \beta_3 W_1^* + \beta_4 W_2^* + \beta_5 V_1^{*2} + \beta_6 V_2^{*2} + \beta_7 W_1^{*2} \\
& + \beta_8 W_2^{*2} + \beta_9 V_1^* V_2^* + \beta_{10} V_1^* W_1^* + \beta_{11} V_1^* W_2^* + \beta_{12} V_2^* W_1^* \\
& + \beta_{13} V_2^* W_2^* + \beta_{14} W_1^* W_2^*,
\end{aligned} \tag{13}
$$

where replacements $x_1 = V_1^*$, $x_2 = V_2^*$, $x_3 = W_1^*$, $x_4 = W_2^*$, $x_5 = V_1^{*2}$, $x_6 = V_2^{*2}$, $x_7 = W_1^{*2}$, $x_8 = W_2^{*2}$, $x_9 = V_1^* V_2^*$, $x_{10} = V_1^* W_1^*$, $x_{11} = V_1^* W_2^*$,

$x_{12} = V_2^* W_1^*$, $x_{13} = V_2^* W_2^*$, $x_{14} = W_1^* W_2^*$, render equation (13) as a simple linear multiple regression as follows:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \beta_6 x_6 + \beta_7 x_7 \\ + \beta_8 x_8 + \beta_9 x_9 + \beta_{10} x_{10} + \beta_{11} x_{11} + \beta_{12} x_{12} + \beta_{13} x_{13} + \beta_{14} x_{14}. \quad (14)$$

All coefficients can be obtained from the simple least-square-error method.

Let us consider the case where we perform n calculations of responses ($n > k$ or $n = k$, where k is the number of variables in equation (14)). The number of unknown coefficients is $p = k+1$. The matrix form of the linear multiple regression model is rewritten as

$$\mathbf{y} = \mathbf{X}\beta + \varepsilon, \quad (15)$$

$$\mathbf{y} = \begin{Bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{Bmatrix}, \quad \mathbf{X} = \begin{bmatrix} 1 & x_{11} & x_{12} & \cdots & x_{1k} \\ 1 & x_{21} & x_{22} & \cdots & x_{2k} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_{n1} & x_{n2} & \cdots & x_{nk} \end{bmatrix},$$

$$\beta = \begin{Bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_k \end{Bmatrix}, \quad \varepsilon = \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{Bmatrix},$$

where $\mathbf{y}$ is a response vector, $\mathbf{X}$ is a variable matrix, β is a coefficient vector, and ε is an error vector. The unbiased estimator $\mathbf{b}$ of coefficient β is obtained using the least-square-error method:

$$\mathbf{b} = (\mathbf{X}^{\mathrm{T}}\mathbf{X})^{-1}\mathbf{X}^{\mathrm{T}}\mathbf{y}. \quad (16)$$

Multiple sets of responses and variables of different points are required to obtain coefficients using the least-square-error-method. To reduce variance of coefficients on the limited number of calculations or experiments, we must carefully select experimental or calculation points. This process is called experimental design. In the present study, a new *D*-optimal laminate is employed [22]: a set of 36 feasible laminates selected from all feasible laminates of 16 plies of balanced symmetric laminates with *D*-optimality.

The *D*-optimal laminates are selected from all possible laminate combinations of 16 plies: 6561 laminates. In the present study, trinary numbers are employed as genes as in Harrison *et al.* [8]; the number of entire laminates is, therefore, $3^8 = 6561$. From the set of candidate laminates, the best set of laminates of 36 is selected with *D*-optimality. The *D*-optimality is a computer-aided design of experiments to reduce variants and co-variants maximizing a determinant of a moment matrix defined as follows [21]:

$$\boldsymbol{M} = \boldsymbol{X}^T \boldsymbol{X}/\boldsymbol{n}. \tag{17}$$

Since the number of unknown coefficients is 15 in the present study, and more than twice the number of unknown coefficients is required empirically, 36 laminates were selected here from feasible laminates with D-optimality. For the objective function of D-optimality, D-efficiency D_{eff} is used; the definition of D_{eff} is shown as follows:

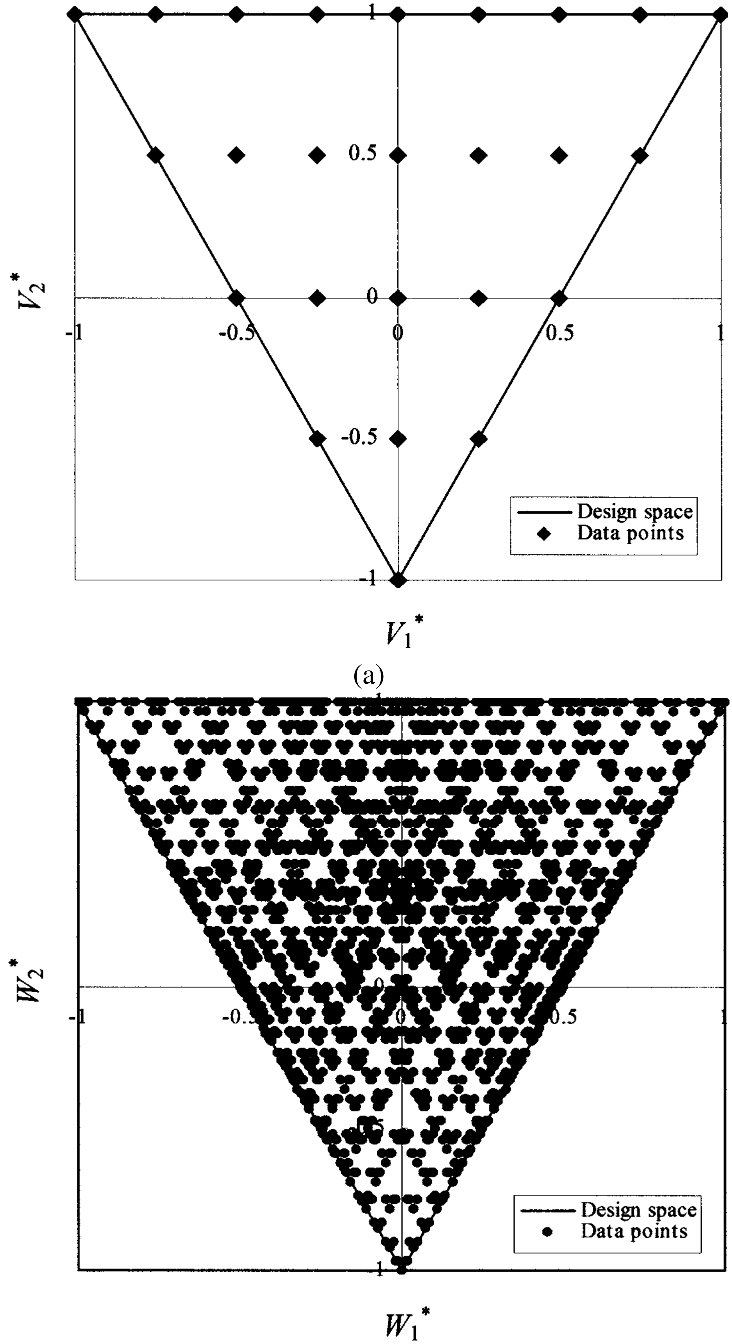
$$D_{eff} = \frac{\left(\mathrm{Det}\left[\mathbf{X}^{\mathrm{T}}\mathbf{X}\right]\right)^{1/p}}{k}. \tag{18}$$

Normalizing all variables x from -1 to 1, the value of D_{eff} must be from 0 to 1. The highest value of D_{eff} gives the best experiment design.

All decimal numbers from 0 to 6560 are coded into trinary numbers to represent chromosome stacking sequences. Only the balance-rule is considered and decoded into practical stacking sequences; the in-plane (V_1^*, V_2^*) and out-of-plane (W_1^*, W_2^*) lamination parameters of these laminates are calculated. These all-feasible laminates are plotted in the in-plane and out-of-plane lamination parameter coordinates as shown in Fig. 2. To avoid calculation difficulties of this quasi-singular matrix, similar laminates whose elements of lamination parameters are within 20% difference with each other are trimmed and 36 laminates are selected. The selected 36 laminates are shown in Table 1.

A remarkable advantage of the D-optimal laminates is that the selected set of laminates can be applied to laminates of any number of plies even though D-optimal laminates are selected from 16-ply laminates (8 genes). Let the number of stacks be $2N$ and consider that ply thickness is changed to $N/8$. A 16-ply laminate has the same total thickness as that of $2N$, and it has lamination parameters that are identical to those of a 16-ply laminate of normal ply thickness. Increased number of stacks does not extend the region of lamination parameters where feasible laminates exist: the region of the set of feasible laminates remains a triangle whose apexes are (1, 1), (-1, 1), and (0, -1) as shown in Fig. 2 in lamination parameter coordinates. Increase of ply number simply causes increased density of plots of feasible laminates in Fig. 2. Increased density simply implies little change of D_{eff} even if we conduct another D-optimal selection from the entire set of candidate laminates of $2N$ for a number larger than $N = 8$. Figure 3 shows D_{eff} change for laminates from 10 plies to 22 plies. In the figure, the abscissa shows the number of stacks ($2N$) and the ordinate shows obtained value of D_{eff} when we conduct laminate selections from candidates of 3^N with D-optimality. Figure 3 shows large change between odd and even quantities of N. When N is an odd number ($2N = 10$, 14, 18, 22), D_{eff} is almost constant at 6.9%; D_{eff} is almost constant at 6.3% when N is an even number ($2N = 12$, 16, 20).

The reason why N is important for D_{eff} is that laminates are symmetric. For example, values of in-plane lamination parameters V_1^* and V_2^* are calculated simply



(b)

Figure 2. Plots of feasible laminates on lamination parameter space (16 plies). (a) In-plane lamination parameter space; (b) bending lamination parameter space.

Table 1.
Selected laminates by lamination D-optimality

1 [	0	0	0	0	0	0	0	0	]s	19 [	45	−45	45	−45	45	−45	0	0	]s
2 [	0	0	0	0	0	0	45	−45	]s	20 [	45	−45	45	−45	45	−45	45	−45	]s
3 [	0	0	0	0	0	90	90	90	]s	21 [	45	−45	45	−45	45	−45	90	90	]s
4 [	0	0	0	0	45	−45	45	−45	]s	22 [	45	−45	45	−45	90	90	90	90	]s
5 [	0	0	0	0	90	90	90	90	]s	23 [	45	−45	90	90	90	0	0	0	]s
6 [	0	0	45	−45	45	−45	45	−45	]s	24 [	45	−45	90	90	90	90	90	90	]s
7 [	0	0	45	−45	45	−45	90	90	]s	25 [	90	0	0	0	0	0	0	0	]s
8 [	0	0	90	90	45	−45	45	−45	]s	26 [	90	45	−45	45	−45	0	0	0	]s
9 [	0	0	90	90	90	90	90	90	]s	27 [	90	45	−45	90	45	90	−45	90	]s
10 [	0	45	−45	45	−45	90	90	90	]s	28 [	90	90	0	0	0	0	0	0	]s
11 [	0	90	0	90	90	90	0	0	]s	29 [	90	90	0	0	0	0	45	−45	]s
12 [	0	90	90	90	90	90	45	−45	]s	30 [	90	90	45	−45	45	−45	0	0	]s
13 [	0	90	90	90	90	90	90	90	]s	31 [	90	90	45	−45	45	−45	45	−45	]s
14 [	45	0	0	−45	0	45	0	−45	]s	32 [	90	90	90	90	0	0	0	0	]s
15 [	45	−45	0	0	0	0	0	0	]s	33 [	90	90	90	90	45	−45	45	−45	]s
16 [	45	−45	0	0	0	0	90	90	]s	34 [	90	90	90	90	90	90	0	0	]s
17 [	45	−45	0	0	0	90	90	90	]s	35 [	90	90	90	90	90	90	45	−45	]s
18 [	45	−45	45	−45	0	0	0	0	]s	36 [	90	90	90	90	90	90	90	90	]s

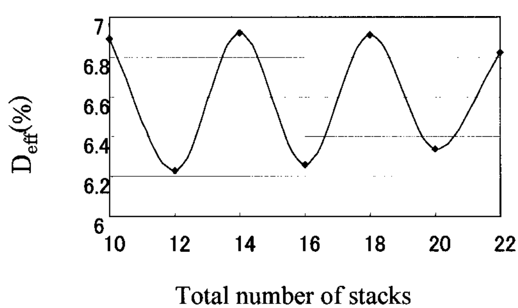


Figure 3. Relation between number of stacks and D_{eff}.

from the volume fraction of fiber angles. This calculation requires division by the number of stacks, implying that values of D_{eff} depend on the value of N.

Figure 3 implies that the change of D_{eff} is very small even if we conduct selections with D-optimality from new candidates of laminates of 3^N. Almost identical values of D_{eff} mean that variations of response surfaces do not change. This implies that the selected set of D-optimal laminates of 16 plies is approximately a better experimental design for laminates of any number of plies simply by changing thickness of each ply.

In the present study, response surfaces are modeled using quadratic polynomials. Useless terms of each response surface are deleted with F-statistics. The quadratic polynomial is adopted because it is simple and design space in lamination parameter coordinates does not have strong non-linearity.

4. IMPLEMENTATION OF GENETIC ALGORITHM

The present study considers a balanced symmetric laminate comprising the set of fiber angles of 0°, ±45°, and 90° plies; the total number of stacks is 16. Therefore, the number of genes of a chromosome is 8 to represent a laminate stacking sequence. As proposed by Harrison *et al.* [8], trinary numbers are employed to represent fiber angles for gene coding.

Decipherment of genes in the chromosome is performed from the leftmost gene; it represents the outermost laminate ply. The trinary number 0 corresponds to a 0° ply and the number 2 corresponds to a 90° ply. First, third, fifth, and odd occurrences of the number 1 correspond to 45° plies, while second, forth and even occurrences of the number 1 correspond to a −45° ply. This implies that an imbalance of 45° plies is equal to a surplus of single 45° plies. A surplus of 45° plies is balanced with the recessive-gene-like strategy proposed by the authors [15]. The four-contiguous-ply rule is also implemented with it. All parameters of the genetic algorithm are tuned by trial and error: the population is 10, the number of generation is 500, probability of crossover is 0.8, and probability of mutation is 0.1875.

5. A NEW CRITERION FOR FITNESS OF RESPONSE SURFACE

5.1. Adjacent laminates

As described before, investigating response-surface approximation accuracy in lamination parameters requires evaluation of adjacent laminates to the provisional optimal laminates obtained with the genetic algorithm; then we must compare values of the fitness function of those adjacent laminates with those of the provisional optimal laminate. It is, however, very difficult to obtain a stacking sequence from a given set of lamination parameters. This causes difficulty in the task of obtaining adjacent laminates around the provisional optimal laminate. To obtain adjacent laminates, we must start from investigations of small changes in lamination parameters by small changes of stacking sequences around the provisional optimal laminate here.

In the in-plane lamination parameters (V_1^*–V_2^*), coordinates (V_1^*, V_2^*) are independent of the order of stacked plies. Coordinates (V_1^*, V_2^*) are decided only with volume fraction of plies. Considering the case of a laminate (total number of plies $= Ns$), the number of 0° plies is $2a$, the number of 45° plies is $2b$ (note that the number of −45°-plies is also $2b$ because the laminate is balanced), and the number of 90° plies is $2c$ ($Ns = 2a + 4b + 2c$; where a, b, and c are non-negative integers). Adjacent laminates and small variations of in-plane lamination parameters indicate the following eight patterns:

(1) $[0_{a+1}/45_{b-1}/-45_{b-1}/90_{c+1}]s$: replace two adjacent ±45° plies with a 0° ply and a 90° ply

$$\Delta V_1^* = 0, \quad \Delta V_2^* = 8/Ns,$$

(2) $[0_{a+2}/45_{b-1}/-45_{b-1}/90_c]s$: replace two adjacent $\pm45^\circ$ plies with two 0° plies

$$\Delta V_1^* = 4/Ns, \quad \Delta V_2^* = 8/Ns,$$

(3) $[0_{a+1}/45_b/-45_b/90_{c-1}]s$: replace a 90° ply with a 0° ply

$$\Delta V_1^* = 4/Ns, \quad \Delta V_2^* = 0,$$

(4) $[0_a/45_{b+1}/-45_{b+1}/90_{c-2}]s$: replace two 90° plies with two adjacent $\pm45^\circ$ plies

$$\Delta V_1^* = 4/Ns, \quad \Delta V_2^* = -8/Ns,$$

(5) $[0_{a-1}/45_{b+1}/-45_{b+1}/90_{c-1}]s$: replace a 0° ply and 90° ply with two adjacent $\pm45^\circ$ plies

$$\Delta V_1^* = 0, \quad \Delta V_2^* = -8/Ns,$$

(6) $[0_{a-2}/45_{b+1}/-45_{b+1}/90_c]s$: replace two 0° plies with two adjacent $\pm45^\circ$ plies

$$\Delta V_1^* = -4/Ns, \quad \Delta V_2^* = -8/Ns,$$

(7) $[0_{a-1}/45_b/-45_b/90_{c+1}]s$: replace a 0° ply with a 90° ply

$$\Delta V_1^* = -4/Ns, \quad \Delta V_2^* = 0,$$

(8) $[0_a/45_{b-1}/-45_{b-1}/90_{c+2}]s$: replace two adjacent $\pm45^\circ$ plies with two 90° plies

$$\Delta V_1^* = -4/Ns, \quad \Delta V_2^* = 8/Ns.$$

These sets of small variations of in-plane lamination parameters are regarded as vectors in in-plane lamination parameter coordinates $(\Delta V_1^*, \Delta V_2^*)$. These eight vectors are shown in the in-plane lamination parameter coordinates in Fig. 4. As shown in this figure, all small variations in in-plane lamination parameter coordinates are classified into eight patterns.

Out-of-plane lamination parameters depend on stacking sequences. Out-of-plane lamination parameters are not independent of in-plane lamination parameters: their relation remains unknown. It is, therefore, very difficult to find the adjacent laminates without simplifying the problem. To simplify the problem, let us find the adjacent laminates from the set of laminates having exactly the same in-plane lamination parameters.

For the case of out-of-plane lamination parameters, we consider a laminate (total number of plies is Ns) similarly to the case of in-plane lamination parameter: the number of 0° plies is $2a$, the number of 45°-plies is $2b$ (note that the number of -45° plies is also $2b$ because the laminate is balanced) and the number of 90° plies is $2c$ ($Ns = 2a + 4b + 2c$; where a, b, and c are non-negative integers).

We consider the case where the ith ply counted from the middle plane of the laminate is replaced with the jth ply. Fiber direction of the ith ply is θ_i and that of

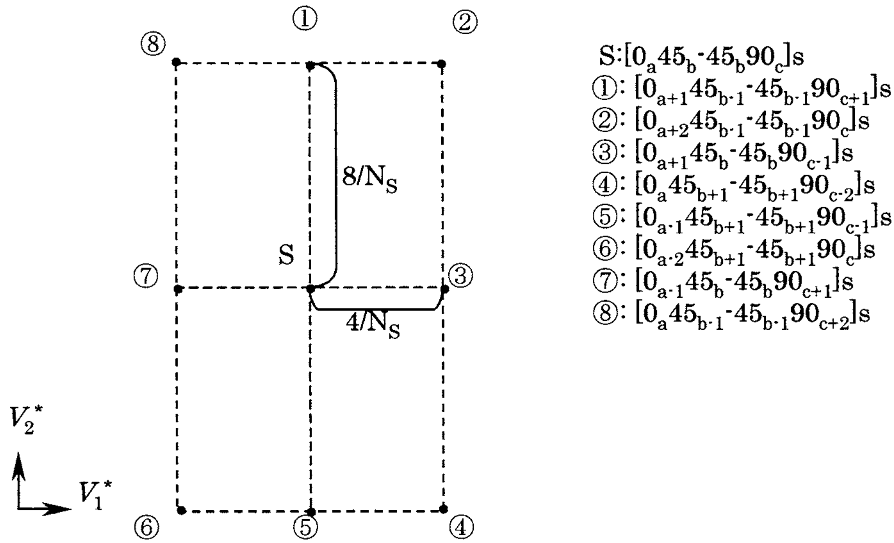


Figure 4. Adjacent laminates on in-plane lamination parameter space.

the jth ply is θ_j $(j > i)$ in this case. Note that replacement of a ply with a ply of the same fiber angle is not considered because it does not make sense; replacement of a 45° ply with −45° ply is also neglected because that does not change out-of-plane lamination parameters of W_1^* and W_2^*; replacement of 45° ply with a 0° ply has the same effect as replacement with a 90° ply. Considering these restrictions, total small variations of the out-of-plane lamination parameters by swapping plies are classified into the six patterns shown in Table 2.

In Table 2, arrows indicate the swap of an outer ply with an inner ply. In the table, the left-side-ply is the outer ply and the right-side-ply is the inner ply. Small changes of out-of-plane lamination parameters $(\Delta W_1^*, \Delta W_2^*)$ of all patterns are also shown in the table. Change of out-of-plane lamination parameters $(\Delta W_1^*, \Delta W_2^*)$ simply means a direction vector of each small variation. Let the number of plies for swapping be represented as i and j $(j > i)$. The most adjacent laminate in each direction is one that minimizes distance g defined as follows.

$$g = \left|\left\{i^3 - (i-1)^3\right\} - \left\{j^3 - (j-1)^3\right\}\right|. \tag{19}$$

Note that the most adjacent laminate is not always obtained with a single swapping. For example, in the case of the stacking sequence of $[0/90/45/-45/90/90/0/0]s$, the most adjacent laminate in the 4th direction of Table 2 (swap of an outer 45° ply with an inner 0° ply) is accomplished through two swaps: replacement of the outermost 0° ply with the third 45° ply from the outermost ply, then replacement of the innermost 0° ply with the fifth −45° ply from the innermost ply. The first swap makes the laminate of $[\underline{45}/90/\underline{0}/-45/90/90/0/0]s$ where the underlined figures represent swapped plies. The second swap makes the laminate of $[45/90/0/\underline{0}/90/90/0/\underline{-45}]s$.

Table 2.
Bending lamination parameter change by swapping

Number of direction	Swapped plies outer ↔ inner	$(\Delta W_1, \Delta W_2)$
1	45 ↔ 90	(0, −2)
2	90 ↔ 45	(−1, 2)
3	0 ↔ 45	(1, 2)
4	45 ↔ 0	(−1, −2)
5	90 ↔ 0	(−2, 0)
6	0 ↔ 90	(2, 0)

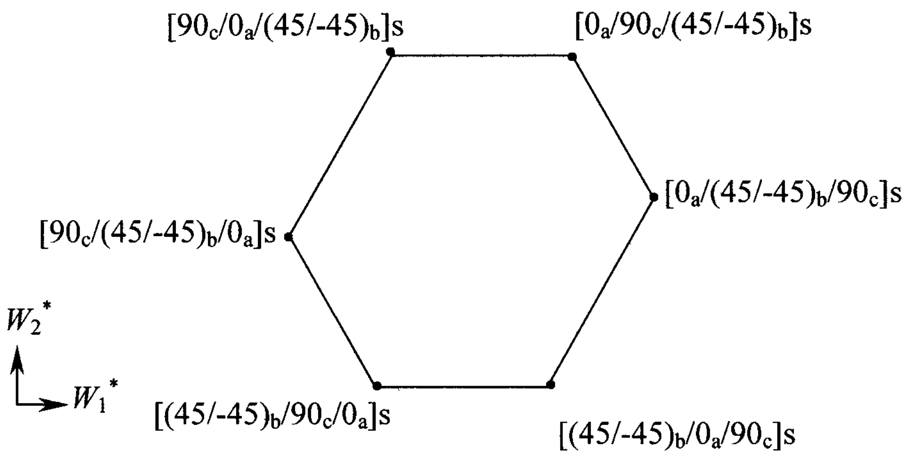


Figure 5. Boundary of feasible laminates on bending lamination parameters of the laminates having the same in-plane lamination parameters.

In general, entire laminates of fixed in-plane lamination parameters can be plotted inside or on the hexagon created using six vectors of maximum distances of g defined in equation (19). Figure 5 shows the hexagon schematic.

5.2. Evaluation criterion

Total number of adjacent laminates calculated from in-plane and out-of-plane lamination parameters is 62 ($= 6 + 8 + 8 \times 6$) when all combinations are counted simply. Since evaluation of 62 cases poses a high cost, a new strategy to reduce calculation cost is indispensable. To reduce the number of evaluations, candidates that are obviously inferior to the provisional optimal laminate are eliminated from evaluations. Inferior candidates can be calculated with the response surface of the objective function. This process is described as follows.

(1) A global response surface is created from responses of laminates selected with D-optimal laminates; the provisional optimal laminate is selected with GA and

real response of the provisional optimal laminate is obtained with FEM analysis (F_{RS}).

(2) FEM analyses are performed for all six adjacent laminates to the provisional optimal laminate obtained by fixing in-plane lamination parameters to those of the provisional optimal laminate (F_{n0j}).

(3) If $\xi F_{RS} < F_{n0j}$, a zoomed response surface is created, and other adjacent laminates in in-plane lamination parameters are compared. All adjacent laminates in in-plane lamination parameter coordinates are collected (note that only the number of plies of each fiber angle can be decided here and the stacking sequence is unknown) and the optimal stacking sequence for each fixed in-plane lamination parameters is selected with GA. From these obtained optimal laminates for each fixed in-plane lamination parameters, FEM analyses are performed for four laminates from the top value of the objective function of the response surface (F_{ni}).

(4) If $\xi F_{RS} > F_{ni}$, all adjacent laminates of the each optimal laminates in the out-of-plane lamination parameter coordinates are prepared; then FEM analyses are performed (F_{nij}).

(5) If $\xi F_{RS} > F_{nij}$, optimality is confirmed:

In this criterion, ξ is practical tolerant error, and ξ is 1.005 for the 0.5% error tolerant case.

Total number of FEM analyses is 38 when all of these procedures are conducted. Most cases are practically 5% error tolerable; the global response surface is sufficient for these cases. The criterion shown here is only for cases requiring a true optimal laminate.

5.3. Application of the method

As an example of application, this method is applied to optimize a stacking sequence for maximizing buckling load and stiffness. Let us consider the case of a symmetric laminate of 16 plies (eight genes). A global response surface obtained from the D-optimal laminates is as follows.

$$f = 0.70302835 + 0.11393678\, V_1^* - 0.396867\, W_1^* - 0.0205731\, W_2^* - 0.2098141\, W_1^{*2} + 0.17454854\, W_2^* W_1^* - 0.0421977\, W_2^{*2}. \tag{20}$$

The adjusted coefficient of multiple determination R^2_{ad} is 0.9716; it seems appropriate that the approximation is obtained from the value of R^2_{ad}.

The GA provides the provisional optimal stacking sequence of [90/45/90/−45/45/−45/0/0]s, which has $(V_1^*, V_2^*, W_1^*, W_2^*) = (0, 0, -0.4219, 0.0469)$. True fitness of the laminate is $F = 0.84430$. After obtaining the provisional optimal laminate, an optimality check is performed for the laminate. Since the provisional optimal laminate has the in-plane lamination parameters of $(V_1^*, V_2^*) = (0, 0)$, adjacent laminates are selected for the optimality checks from the set of laminates

Table 3.
Adjacent laminates on the plane of $W_1^* - W_2^*$ ($V_1^* = 0$, $V_2^* = 0$)

j		V_1^*	V_2^*	W_1^*	W_2^*	F_{n0i}
1	[90/45/−45/90/45/−45/0/0]s	0	0	−0.4336	−0.0703	0.82691
2	[90/90/45/−45/45/−45/0/0]s	0	0	−0.5625	0.1875	0.86511
3	[90/45/90/−45/45/0/−45/0]s	0	0	−0.4688	−0.0938	0.83314
6	[90/45/0/−45/45/−45/90/0]s	0	0	−0.4336	−0.0703	0.71762

having exactly the same in-plane lamination parameters as $(V_1^*, V_2^*) = (0, 0)$. True fitness values for all these laminates are shown in Table 3.

The adjacent laminate of [90/90/45/−45/45/−45/0/0]*s* has superior fitness of $F = 0.86511$ to fitness of the provisional optimal laminate. This means that the provisional optimal laminate is not the true optimal and a zoomed response surface is required to obtain the true optimal laminate.

The zoomed response surface is created from the 36 D-optimal laminates reselected from the partial set of candidate laminates: the partial set of laminates is almost a third of all laminates, and candidates are selected with distance (L) from the provisional optimal laminate in the lamination parameter coordinates defined as

$$L = \sqrt{\left(V_1^* - V_1^{*0}\right)^2 + \left(V_2^* - V_2^{*0}\right)^2 + \left(W_1^* - W_1^{*0}\right)^2 + \left(W_2^* - W_2^{*0}\right)^2}, \tag{21}$$

where V_1^{*0}, V_2^{*0}, W_1^{*0} and W_2^{*0} are lamination parameters of the provisional optimal laminate obtained from the global response surface. The partial set of candidate laminates is determined as follows: the total number of practical laminates satisfying the balance rule for symmetric laminates of 16 plies is 3281; the partial set is approximately a third of the entire number of laminates, which means the number of laminates is approximately 1100 for the partial set; $L = 0.76$ is selected because the partial set of laminates satisfying $L < 0.76$ is 1106. Selected 36 D-optimal laminates are shown in Table 4.

The zoomed response surface created as mentioned is the following.

$$\begin{aligned} f = {} & 0.66382444 + 0.0888221\, V_1^* + 0.01807134\, V_2^* - 0.4266983\, W_1^* \\ & - 0.0290668\, W_2^* - 0.0365956\, V_2^* V_1^* - 0.0776761\, W_1^* V_1^* \\ & - 0.1110584\, W_1^{*2} + 0.03978906\, W_2^* V_1^* + 0.0708057\, W_2^* W_1^* \\ & - 0.0370273\, W_2^{*2}. \end{aligned} \tag{22}$$

The value of R_{ad}^2 is 0.9721. The value of R_{ad}^2 is similar to that of the global response surface shown before. This means that R_{ad}^2 does not indicate the degree of approximation of a response surface of the entire region of the design space.

With GA using the zoomed response surface of equation (22) as a fitness function, provisional optimal laminate and practical optimal laminates are searched; these

Table 4.
Selected laminates for zooming by lamination D-optimality

1	[90	90	45	−45	45	−45	0	0]s	19	[45	0	−45	90	45	−45	0	0]s
2	[45	90	90	0	−45	45	−45	90]s	20	[0	45	−45	90	45	−45	90	90]s
3	[90	45	−45	45	−45	0	0	0]s	21	[45	−45	90	90	45	−45	45	−45]s
4	[45	90	−45	90	90	45	90	-45]s	22	[90	90	0	90	45	−45	45	−45]s
5	[45	−45	90	90	90	0	0	0]s	23	[45	−45	0	90	90	90	0	90]s
6	[90	90	45	−45	45	−45	45	−45]s	24	[45	90	−45	0	90	90	90	90]s
7	[90	45	−45	45	−45	45	−45	0]s	25	[90	0	45	0	−45	0	45	−45]s
8	[90	0	45	−45	45	−45	45	−45]s	26	[45	90	90	90	90	−45	90	0]s
9	[90	90	45	−45	0	0	0	0]s	27	[45	−45	90	45	90	−45	90	90]s
10	[45	−45	90	90	0	0	0	0]s	28	[90	90	45	90	−45	90	0	0]s
11	[45	90	0	−45	45	−45	45	−45]s	29	[90	90	45	90	90	−45	45	−45]s
12	[90	90	45	0	−45	0	0	0]s	30	[90	90	90	45	−45	45	90	−45]s
13	[90	45	0	90	0	−45	90	90]s	31	[0	45	−45	45	90	90	90	−45]s
14	[90	90	0	0	45	−45	45	−45]s	32	[45	0	−45	45	90	−45	90	0]s
15	[0	90	90	45	90	−45	45	−45]s	33	[90	0	0	45	−45	45	90	−45]s
16	[45	−45	90	45	−45	0	0	0]s	34	[90	45	0	−45	0	0	0	90]s
17	[45	−45	45	90	90	−45	90	0]s	35	[45	0	90	90	0	−45	0	90]s
18	[45	−45	90	90	90	90	90	0]s	36	[0	90	45	−45	0	45	0	−45]s

Table 5.
Optimization results using zoomed response surface

	V_1^*	V_2^*	W_1^*	W_2^*	$\overline{F}$	F
[90/90/45/90/−45/0/0/0] s	0	0.5	−0.6445	0.5	0.8551	0.83889
[90/90/45/−45/45/−45/0/0]s	0	0	−0.5625	0.1875	0.8544	0.86511
[90/90/45/−45/90/0/0/0]s	0	0.5	−0.5977	0.4063	0.8531	0.86158
[90/45/90/90/−45/0/0/0]s	0	0.5	−0.5742	0.3593	0.8514	0.87291

results are shown in Table 5. In Table 5, lamination parameters of four laminates and values of fitness functions are shown: the value obtained from the zoomed response surface ($\overline{F}$) and the true value obtained from analysis (F).

Optimality of the provisional optimal laminate obtained with the zoomed response surface is checked with the optimality criterion discussed previously. First, adjacent laminates in out-of-plane lamination parameters of fixed in-plane lamination parameters $(V_1^*, V_2^*) = (0, 0.5)$ are selected. Selected laminates are shown in Table 6 with values F of actual analyses. The table represents that the provisional optimal laminate is superior to all adjacent laminates.

Second, four laminates adjacent to the provisional laminate in in-plane lamination parameters are selected to check optimality in the in-plane lamination parameter coordinates. The four laminates are selected from candidate laminates adjacent to provisional laminates using the zoomed response surface; actual analyses of the objective function are performed for laminates. Results are shown in Table 7.

Table 6.
Adjacent laminates on the plane of $W_1^* - W_2^*$ ($V_1^* = 0$, $V_2^* = 5$)

j		V_1^*	V_2^*	W_1^*	W_2^*	F_{n0j}
1	[90/45/90/−45/90/0/0/0]s	0	0.5	−0.5727	0.2656	0.85970
2	[90/90/45/90/−45/0/0/0]s	0	0.5	−0.6445	0.5	0.83890
3	[90/45/90/90/0/−45/0/0]s	0	0.5	−0.5391	0.4297	0.85580
6	[90/45/90/0/−45/90/0/0]s	0	0.5	−0.4102	0.3594	0.81024

Table 7.
Adjacent laminates on the plane of $V_1^* - V_2^*$

i		V_1^*	V_2^*	W_1^*	W_2^*	$\overline{F}$	F_{ni}
1	[90/90/45/−45/45/−45/0/0]s	0	0	−0.5625J	0.1875	0.8651	0.86511
2	[90/90/45/−45/0/0/0/0]s	0.25	0.5	−0.4531	0.4063	0.8429	0.84807
3	[90/90/45/−45/90/90/0/0]s	−0.25	0	−0.6719	0.4063	0.8374	0.83832
4	[90/45/−45/90/90/90/0/0]s	−0.25	0.5	−0.5428	0.1484	0.8217	0.84766

Table 8.
Adjacent laminates on the plane of $W_1^* - W_2^*$ ($V_1^* = 0$, $V_2^* = 0$)

j		V_1^*	V_2^*	W_1^*	W_2^*	F_{1j}
1	[90/45/90/−45/45/−45/0/0]s	0	0	−0.4922	0.0469	0.84430
3	[90/90/45/−45/45/0/−45/0]s	0	0	−0.5039	0.3047	0.83721
6	[90/0/45/−45/45/−45/90/0]s	0	0	−0.0938	0.1875	0.68414

Table 9.
Adjacent laminates on the plane of $W_1^* - W_2^*$ ($V_1^* = 0.25$, $V_2^* = 0.5$)

j		V_1^*	V_2^*	W_1^*	W_2^*	F_{2j}
1	[90/45/90/−45/0/0/0/0]s	0	0	−0.3828	0.2656	0.82726
3	[90/90/45/0/−45/0/0/0]s	0.25	0.5	−0.4063	0.5	0.82575

Table 7 shows conclusively that the provisional optimal laminate is superior to the four adjacent laminates.

Third, for each laminate shown in Table 7, adjacent laminates in the out-of-plane lamination parameters of the fixed in-plane lamination parameters of each laminate are obtained to evaluate optimality; actual analyses are performed for all these laminates. Results are shown in Tables 8 to 11. All these laminates indicate that the provisional optimal laminate is superior to adjacent laminates. This confirms optimality of the provisional optimal laminate obtained from the zoomed response surface.

Table 10.
Adjacent laminates on the plane of $W_1^* - W_2^*$ ($V_1^* = -0.25$, $V_2^* = 0$)

j		V_1^*	V_2^*	W_1^*	W_2^*	F_{3j}
1	[90/45/90/90/−45/90/0/0]s	−0.25	0	−0.6484	0.3594	0.84967
2	[90/90/45/90/−45/90/0/0]s	−0.25	0.5	−0.1719	0.5	0.81563
6	[90/90/45/-45/90/0/90/0]s	−0.25	0.5	−0.625	0.4063	0.83826

Table 11.
Adjacent laminates on the plane of $W_1^* - W_2^*$ ($V_1^* = -0.25$, $V_2^* = 0.5$)

j		V_1^*	V_2^*	W_1^*	W_2^*	F_{4j}
1	[45/90/90/−45/90/90/0/0]s	−0.25	0.5	−0.5195	0.1016	0.84072
2	[90/45/90/−45/90/90/0/0]s	−0.25	0.5	−0.6016	0.2656	0.86450
3	[90/45/0/90/90/90/−45/0]s	−0.25	0.5	−0.2969	0.3125	0.74526
6	[90/45/-45/90/90/0/90/0]s	−0.25	0.5	−0.4961	0.1484	0.82956

Finally, the optimal laminate is confirmed by checking all feasible laminates; the obtained optimal laminate is confirmed to be the true optimal laminate. Total number of analyses for this method in this example is 100. Admitting an error of 5%, it is obvious that the optimal laminate obtained from the global response surface is sufficient.

6. CONCLUDING REMARKS

For stacking sequence optimizations using a genetic algorithm with response surface in lamination parameters, a new criterion to judge optimality of stacking sequences is proposed here; that method is applied to a stacking sequence optimization problem to maximize buckling load and stiffness. In the case that 5% error is admitted, a global response surface is sufficient. However, the present method provides the real optimal laminate with 100 analyses to obtain the real optimal stacking sequence.

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