Identifying Delamination in Cross-ply and Quasi-isotropic Beams of CFRP by a Standardized Electrical Resistance Method

Akira Todoroki[*1], **Yuuki Tanaka**[*2] **and Yoshinobu Shimamura**[*3]
[*1]Department of Mechanical Sciences and Engineering, Tokyo Institute of Technology, 2-12-1, O-okayama, Meguro, Tokyo 1528552, Japan. E-mail: atodorok@ginza.mes.titech.ac.jp
[*2]Graduate student of Tokyo Institute of Technology
[*3]Department of Mechanical Sciences and Engineering, Tokyo Institute of Technology, 2-12-1, O-okayama, Meguro, Tokyo 1528552, Japan

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SUMMARY

An electrical resistance change method is widely accepted for the identification of damage in electrically conductive composites. To identify the delamination location more precisely, a new electrical resistance change method involving the standardization of measured electrical resistance changes has been proposed. In the present paper, the new method was applied to cross-ply and quasi-isotropic laminated beams, and the effectiveness of the method was confirmed experimentally. Comparing the results for two different beams, the applicability of the method was confirmed. Moreover, the effect of the stacking sequence was investigated experimentally. Although the standardized electrical resistance changes of the two stacking sequences resembled each other when the delamination location and length were similar, response surfaces to estimate delamination location must be made individually to obtain reliable results.

INTRODUCTION

Recently many researchers have adopted an electrical resistance change method to detect internal damage such as fiber breakages, delaminations and matrix cracks in CFRP (Carbon Fiber Reinforced Plastics) laminates[1-15]. In contrast to fiber-optic sensors, the electrical method does not require expensive instrumentation. Since the method uses the carbon fibers themselves as sensors for damage identification, it does not reduce the static strength or fatigue strength, and it is applicable to existing structures merely by attaching multiple electrodes to the target CFRP structures.

The authors' research group has investigated the applicability of the electrical resistance change method by various experiments[16-18]. Orthotropic electrical conductivity was also measured experimentally for three types of fiber volume fractions[19], showing that the electrical conductivity in the thickness direction of CFRP was approximately one thousandth of the conductivity in the fiber direction. This large difference creates difficulties in the identification of delamination locations.

For these studies, delamination location and length were obtained from the measured electrical resistance changes of multiple segments between electrodes. This is one of the inverse problems. To solve the inverse problem, a response surface method was adopted instead of using artificial neural networks: a response is approximated with polynomials. Estimations of the delamination location were inadequate compared to those for delamination length. It was also necessary to improve estimation performance, especially for small delaminations[17,18].

The previous study[20] used FEM analyses of various delamination lengths and locations in beam-type specimens to investigate the reason for the large estimation errors in delamination locations; furthermore an improvement in higher estimation performance was proposed for the data standardization method before making response

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[*1] Corresponding author.

surfaces: all electrical resistance changes are standardized by the each length of the measured electrical resistance change vector. The standardization method, however, magnifies not only the electrical resistance change (due to the formation of a delamination crack) but also various experimental errors. These experimental errors are induced by delamination crack surface contact, matrix cracking and electromagnetic noise, and they are magnified by standardization. The applicability of a new method using standardization, therefore, must be investigated experimentally against experimental errors. This is the first objective of the present study.

The second objective is to investigate the effect of stacking sequences on the identification of delamination location. The present study deals with the delamination monitoring of beam specimens with two types of stacking sequence: cross-ply laminates and quasi-isotropic laminates. A delamination crack is made in the beam specimen with a short beam shear test, and electrical resistance changes are then measured using bridge circuits. The results are standardized and response surfaces are made from the standardized data in order to estimate the delamination location. The difference between the two stacking-sequence types is discussed and their results are compared.

DELAMINATION MONITORING SYSTEM

Principle of the Electrical Resistance Change Method

Carbon fibers have a high electrical conductivity, and the epoxy matrix is an insulator. For ideal carbon/epoxy composites, the electrical conductivity in the fiber direction is high. The ideal conductivity can be easily calculated by multiplying the fiber volume fraction by the electrical conductivity of carbon fibers. On the other hand, the electrical conductivity in the transverse direction drops to almost zero for ideally straight aligned fibers.

A carbon fiber in a unidirectional ply is not straight. Curved carbon fibers contact each other, and that makes a carbon-fiber network in a ply. The contact-network brings non-zero electrical conductivity even in the transverse direction. In the same way, the fiber-network produces non-zero electrical conductivity in the thickness direction in a ply. The electrical conductivity in the transverse direction is much lower than that in the fiber direction. As a result, CFRP laminates have high orthotropic electrical resistance. Although the fiber-network structure in the thickness direction is almost the same as it is in the transverse direction, through-the-thickness conductivity is lower than the transverse conductivity for normal laminates. This is because thin resin-rich interlaminar layers exist and they are electrically insulating. The contact between plies causes non-zero electrical conductivity in the thickness direction. The breakage of the contact network increases the electrical resistance of the carbon/epoxy composites. Therefore, delamination cracks can be detected by electrical resistance changes in carbon/epoxy composite laminates.

Schematic Model of Monitoring System

Figure 1 is a schematic representation of a delamination-monitoring system proposed by the authors. Multiple electrodes are mounted on a beam-type specimen surface, as shown in Figure 1. All these electrodes are placed on one side of the specimen. (Usually it is impossible to place electrodes and lead wires outside aircraft structures. The location of the electrodes on one side is a model for placing electrodes in a thin shell aircraft structure.)

The authors have performed several FEM analyses and shown that the electric current should be changed in the fiber direction of the surface ply to monitor a delamination crack[20]. Electrical resistance changes for each segment between electrodes are measured with a conventional electrical resistance bridge circuit. The electrical resistance changes between all of the segments are measured for various locations and lengths of delaminations. Using the measured electrical resistance changes, a relationship between electrical resistance change and delamination length is obtained using a response surface. The measured electrical resistance changes are standardized by means of the norm of a vector comprising these electrical resistance changes. The standardized electrical resistance changes are used for making a response surface to estimate delamination locations.

Identification of Delamination Location and Length

Let us consider Figure 1. Four electrical resistance changes are measured at four segments (electrical resistance change: E_1, E_2, E_3 and E_4). To make the response surfaces, many experiments or calculations must be performed. From the experiments, several data sets of delamination location, length and electrical resistance changes are obtained. In the case where quadratic polynomials are adopted, the response method to experimental data. This is one of the two objectives of the present paper.

EXPERIMENTAL METHOD

Specimens

The material used in the present paper was unidirectional carbon/epoxy prepreg. It was A125-Rc33% produced by the Shin-Nihon Steel Chemistry Co. The stacking sequence of the cross-ply laminate was [0/90/0/90]s and that of the quasi-isotropic one was [0/45/-45/90]s. The thickness was approximately t=1 mm. The cure schedule was 180°C at 1MPa for 2 h. To measure the electrical resistance changes between electrodes using a two-probe method, reliable electrodes are indispensable. To make the electrodes, a copper foil rectangle 0.02 mm thick was mounted on the prepreg laminates; the electrodes were then co-cured with the laminate. Beam-type specimens 15 mm wide were made from laminated plates, as shown in Figure 2. The spacing between the electrodes was 45 mm.

Measurement of Electrical Resistance Change

Electrical Circuit

Since the electrical resistance change due to delamination was very small, it was measured by the electrical-resistance-bridge circuit shown in Figure 3. This circuit was similar to that used with conventional strain gauges. Therefore, conventional strain-gauge

Figure 2 Specimen configuration

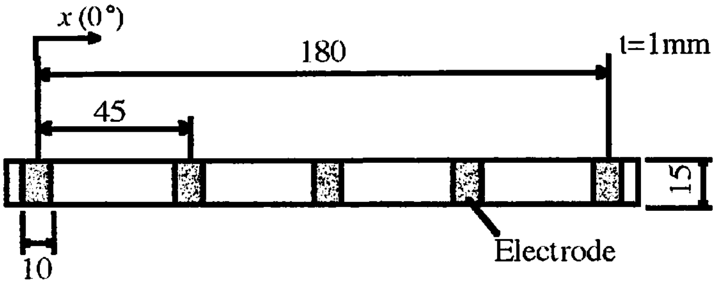


Figure 3 Electrical resistance bridge circuit

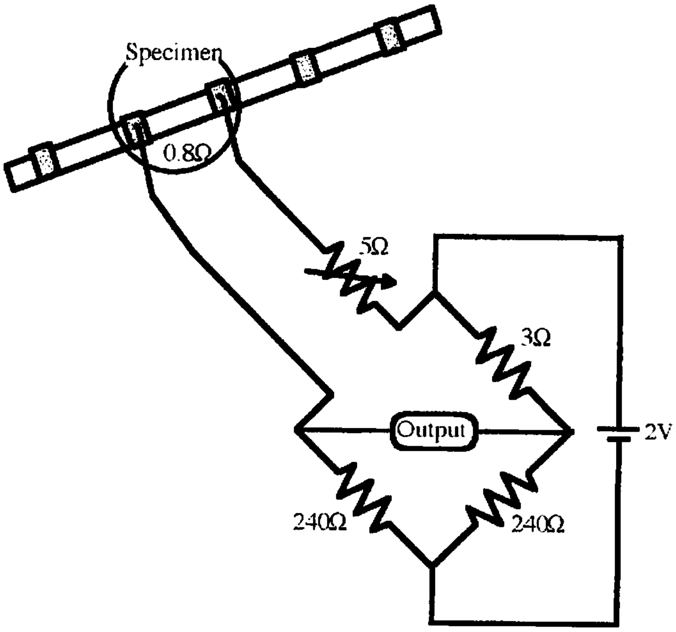
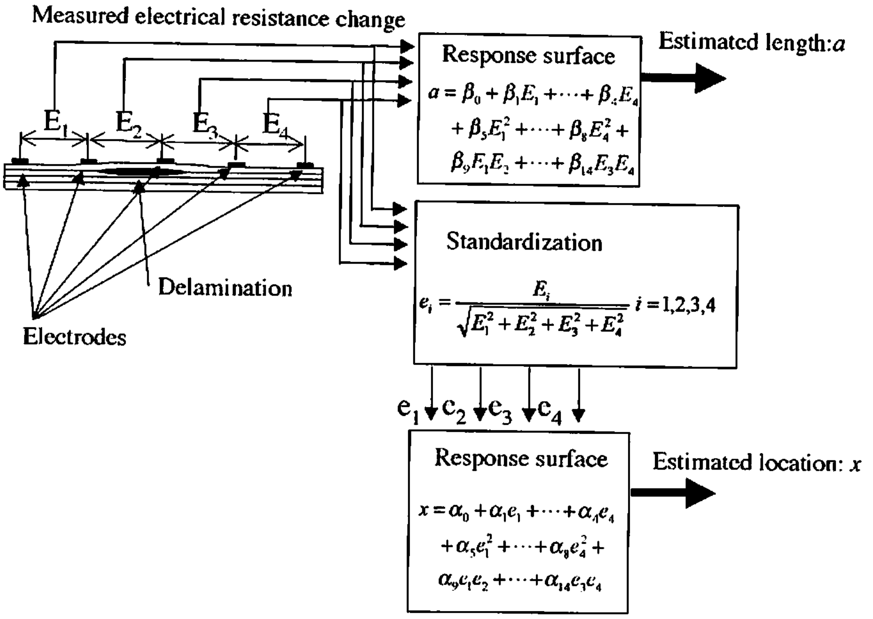


Figure 1 Schematic image of delamination monitoring system using electrical resistance change method



surfaces to estimate the delamination location (x) and length (a) are as follows.

$$x=\beta_0^x+\beta_1^xE_1+\beta_2^xE_2+\beta_3^xE_3+\beta_4^xE_4+\beta_5^xE_1^2+\beta_6^xE_2^2+\beta_7^xE_3^2+\beta_8^xE_4^2+\beta_9^xE_1E_2+\beta_{10}^xE_1E_3+\beta_{11}^xE_1E_4+\beta_{12}^xE_2E_3+\beta_{13}^xE_2E_4+\beta_{14}^xE_3E_4 \tag{1}$$

$$a=\beta_0^a+\beta_1^aE_1+\beta_2^aE_2+\beta_3^aE_3+\beta_4^aE_4+\beta_5^aE_1^2+\beta_6^aE_2^2+\beta_7^aE_3^2+\beta_8^aE_4^2+\beta_9^aE_1E_2+\beta_{10}^aE_1E_3+\beta_{11}^aE_1E_4+\beta_{12}^aE_2E_3+\beta_{13}^aE_2E_4+\beta_{14}^aE_3E_4 \tag{2}$$

where all the coefficients (β^x_i and β^a_i, i=1...14) are obtained by the least square error method[21]. The lack of fit of the response surface can be evaluated with an adjusted coefficient of multiple determination R_{adj}^2 (see reference [21]). R_{adj}^2 is defined as follows.

$$R_{adj}^2=1-\frac{SS_E/(n-k-1)}{S_{yy}/(n-1)} \tag{3}$$

where SS_E is the square sum of errors, S_{yy} is the total sum of squares, n is the number of data sets and k is the number of unknown coefficients. The R_{adj}^2 is equal to or lower than 1.0. A high value of R_{adj}^2 implies a good fit. When the response surface shows a very good fit, the value of R_{adj}^2 approaches unity.

As far as the estimation of delamination locations is concerned, the response surface of equation (2) did not provide good estimations[17,18,20]. The measured electrical resistance changes were standardized with a norm as follows.

$$e_i=\frac{E_i}{\sqrt{E_1^2+E_2^2+E_3^2+E_4^2}},\quad (i=1,2,3,4) \tag{4}$$

Using these standardized electrical resistance changes, the delamination location was estimated as follows.

$$x=\alpha_0^x+\alpha_1^xe_1+\alpha_2^xe_2+\alpha_3^xe_3+\alpha_4^xe_4+\alpha_5^xe_1^2+\alpha_6^xe_2^2+\alpha_7^xe_3^2+\alpha_8^xe_4^2+\alpha_9^xe_1e_2+\alpha_{10}^xe_1e_3+\alpha_{11}^xe_1e_4+\alpha_{12}^xe_2e_3+\alpha_{13}^xe_2e_4+\alpha_{14}^xe_3e_4 \tag{5}$$

where all the coefficients (α^x_i, i=1...14) were obtained by the least square error method.

The standardized electrical resistance change method was applied to the FEM analyses in the previous study[20], and the method provided excellent estimations of delamination locations. The FEM analyses, however, do not include experimental errors. The standardization method magnifies the characteristics of the measured electrical resistance change, but it also magnifies the experimental errors. The method must be confirmed by applying the

Measured Electrical Resistance Changes

Typical measured electrical resistance changes in both specimens are shown in Figure 6. The abscissa is the number of segments between electrodes, and the ordinate is the measured fraction of electrical resistance change divided by the gauge factor. These two specimens had similar delamination cracks. For the cross-ply specimen (triangle symbols), the measured delamination length was 5.2 mm and the location was 115 mm (segment #3). For the quasi-isotropic specimen (circle symbols), the measured delamination length was 5.0 mm and the location was 113 mm (segment #3). The measured fraction of the electrical resistance was similar in both cases as shown in Figure 6.

Figure 7 shows the standardized fraction of the electrical resistance change for the two cases. For the cross-ply specimen (triangle symbols), the measured delamination length was 11.2 mm and the location was 155 mm (segment #4). For the quasi-isotropic specimen (circle symbols), the measured delamination length was 11.4 mm and the location was 153 mm (segment #4). Although these delamination cracks were similar, the measured fractions of the electrical resistance changes were very different as shown in Figure 7.

These measured fractions of the electrical resistance changes included experimental errors, and there were also other sources of error related to the delamination crack surface contact, plastic deformation of angle plies in the quasi-isotropic laminates and damage in the other plies. These error factors may have been responsible for the difference in the fractions of the electrical resistance changes in Figure 7. So far, the cause of the difference is not clear.

Identification of Delaminations with Conventional Response Surfaces

Figure 8 shows the results of our attempt to identify the delamination location in the cross-ply specimen, and Figure 9 shows the results for delamination length. In Figure 8, the abscissa is the measured delamination location and the ordinate is the estimated location with the response surface without using standardizations of the measured electrical resistance changes. In Figure 9, the abscissa is the measured delamination length and the ordinate is the estimated

Figure 6 Measured fractions of the electrical resistance changes of two specimens. Cross-ply (triangle symbol) has a delamination of 5.2 mm length at a distance of 115 mm from edge. Quasi-isotropic (Circle symbol) has a delamination of 5.0 mm at a distance of 113 mm from the edge

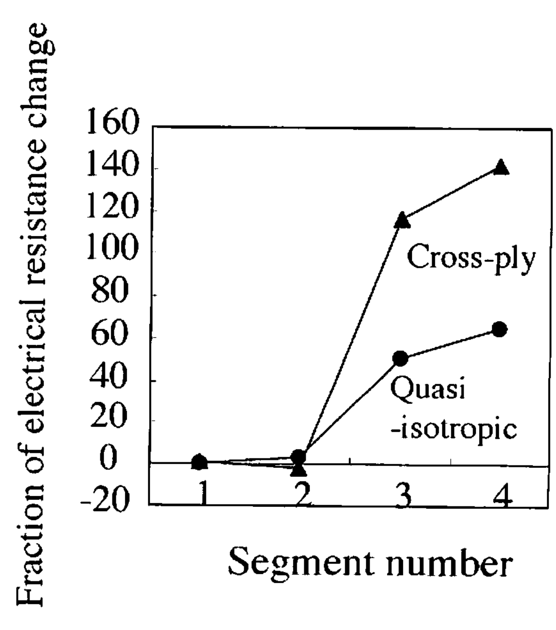
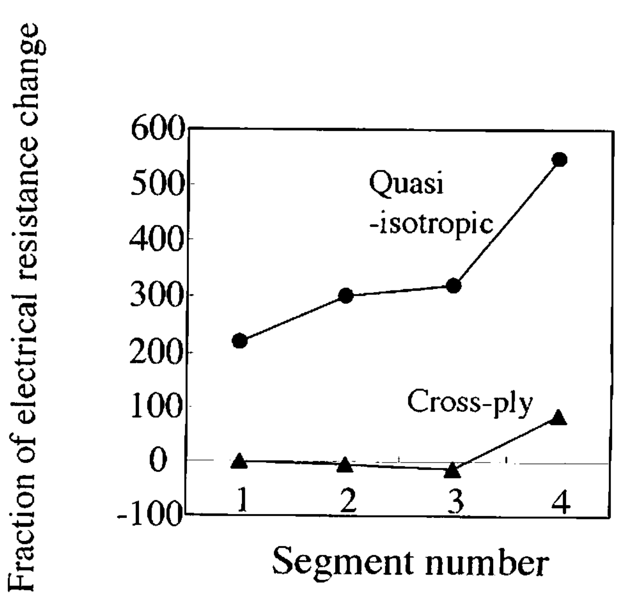


Figure 7 Measured fractions of the electrical resistance changes of two specimens. Cross-ply (triangle symbol) has a delamination of 11.2 mm length at a distance of 155 mm from the edge. Quasi-isotropic (Circle symbol) has a delamination of 11.4 mm at a distance of 153 mm from the edge



amplifiers can be used to measure the electrical resistance change of the specimens. This leads to the labeling of the instrument output as "strain", but it does not imply specimen deformation. The output "strain ε" means the fraction of the electrical resistance change divided by a gauge factor: $\varepsilon=\Delta R/(R_0K)$, where ΔR is the electrical resistance change, R_0 is the initial electrical resistance and K is a gauge factor.

Experimental Procedures

To create a delamination crack in each beam-type specimen, an interlaminar-shear test was employed. The middle point was loaded from the opposite side to the electrode mountings. (Since the specimen was a thin laminate, loading made a large delamination crack in the 0°-90° interface.) After making the delamination crack, the electrical resistance changes of all the segments between the electrodes were measured using a conventional strain-amplifier. Delamination location and length were measured using an ultrasonic C-scan image. The delamination location was decided at the center point of the delamination crack from the specimen end. Delamination length was defined as the maximum of the projected length of the delamination in the longitudinal direction. The total number of experiments with the cross-ply specimens was 23 and with the quasi-isotropic specimens was 21.

RESULTS AND DISCUSSION

Observed Configuration of Delamination

Typical measured delamination crack configurations of cross-ply laminates and quasi-isotropic laminates are shown in Figures 4 and 5 respectively. Pictorial images of C-scans of both specimens are shown here. White areas denote delamination cracks, and black lines in the white areas mean matrix cracks. Since the maximum shear stress occurs in the middle of the specimen in the thickness direction, delamination cracks occur made in the middle of the beam in the thickness direction. For the cross-ply laminates, the delamination crack was found at the interface between the 0°-ply and the 90°-ply in the middle of the specimen in the thickness direction. For the quasi-isotropic laminates, the delamination crack occurs at the middle interface between the 90°-ply and the -45°-ply. These delamination cracks always accompany matrix cracks and a transition crack between interfaces as shown in Figures 4 and 5. Since each type of specimen has a delamination crack in the middle plies, the distance from the surface where electrodes are mounted is always the same. This means the difference in electrical resistance changes between the two types comes solely from the difference in the ply angles when the delamination length is the same.

Figure 4 Definition of delamination location and size with an image of ultrasonic C-scan. Typically a delamination of a cross-ply laminate is located in the middle of the laminate with a matrix crack

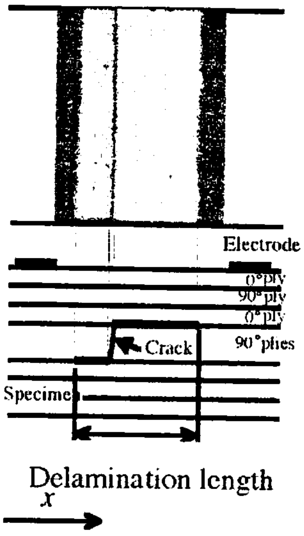
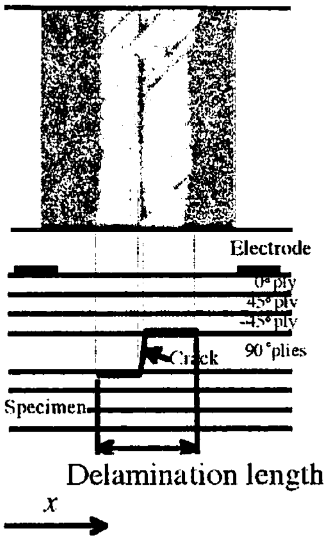


Figure 5 Definition of delamination location and size with an image of ultrasonic C-scan. Typically a delamination of a quasi-isotropic laminate locates in the middle of the laminate with a matrix crack



Empirically, when the value of R_{adj}^2 is larger than 0.8 or 0.85, we can judge whether the response surface gives a good fit. The values of R_{adj}^2 of the response surfaces for delamination locations of both types were not good enough. In contrast, the response surfaces for the delamination length provided better values. The poor estimations of delamination location without standardizations were also observed in FEM analyses[20], and even in the experimental results.

Figure 12 reveals the estimation results for the cross-ply specimens with the standardizations. Figure 13 shows the estimation results for the quasi-isotropic specimens with the standardizations. They show that the errors in estimations were significantly reduced in both specimens. The values of R_{adj}^2 of the response surfaces for delamination locations of both types were 0.96 for the cross-ply specimens and 0.99 for the quasi-isotropic ones.

These high values of R_{adj}^2 reveal that the standardization method is quite efficient even for the experimental data with inevitable experimental errors. As mentioned before, electrical resistance changes obtained experimentally include experimental errors, such as the effect of delamination surface contacts and plastic deformation of off-axis plies. This good result means that these factors do not affect the estimation performance of the electrical resistance change method with standardizations for beam type specimens. The method should be applied to plate type specimens and this will be addressed in our future work.

Effect of Stacking Sequences

Figures 14 and 15 show the results of the standardized electrical resistance changes of Figure 6 and Figure 7 respectively. In the case of Figure 14, the two results of the standardized electrical resistance were almost identical. In the case of Figure 15, however, they two were different. Most of the standardized results resembled those of Figure 14 and situations like that shown in Figure 15 were rare. As mentioned before, the distance in the thickness direction from the surface where the electrodes were mounted was the same for both types of specimen. This explains the similarity in the standardized electrical resistance changes in most of the specimens, although there are some exceptions.

Figure 12 Estimated delamination location with data standardization, for cross-ply laminates

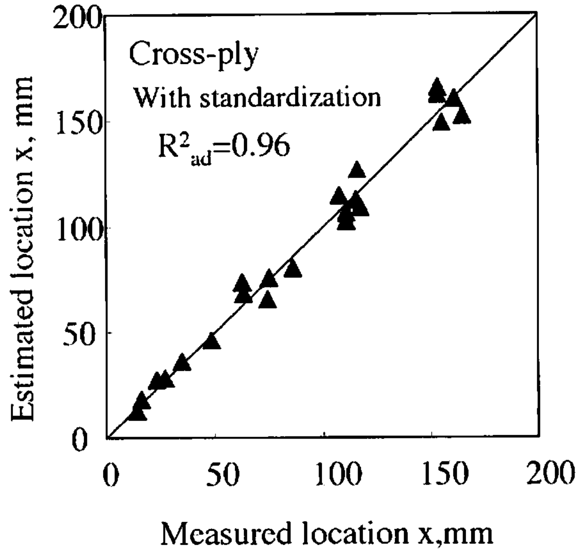
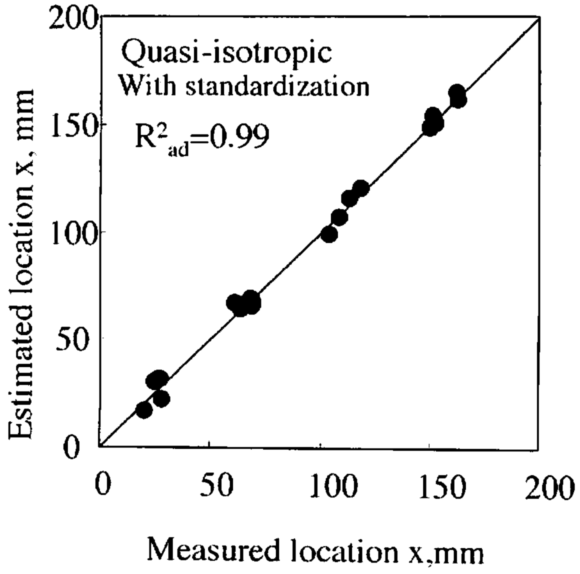


Figure 13 Estimated delamination location with data standardization, for quasi-isotropic laminates



In order to investigate the effect of the difference in the standardized electrical resistances, the standardized electrical resistance changes of the quasi-isotropic specimens were input into the response surface for the estimation of delamination location of cross-ply specimens. The estimated delamination locations are shown in Figure 16. The abscissa is the measured delamination location of the quasi-isotropic specimens, and the ordinate is the estimated results obtained from the response surface of the cross-ply

Figure 8 Estimated delamination location without using data standardization, for cross-ply laminates

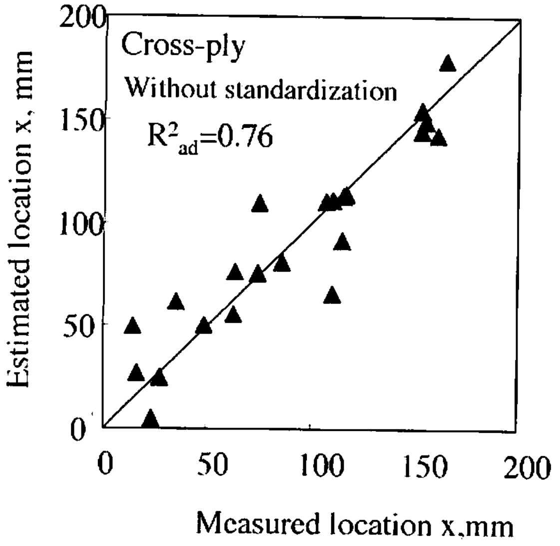


Figure 9 Estimated delamination length of cross-ply laminates

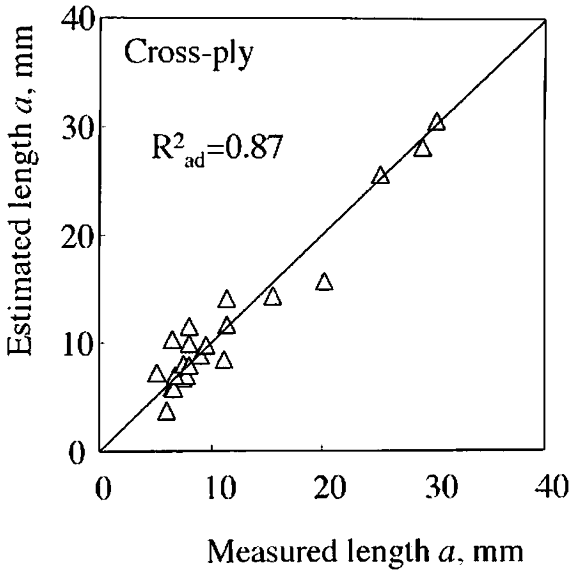


Figure 10 Estimated delamination location without using data standardization, for quasi-isotropic laminates

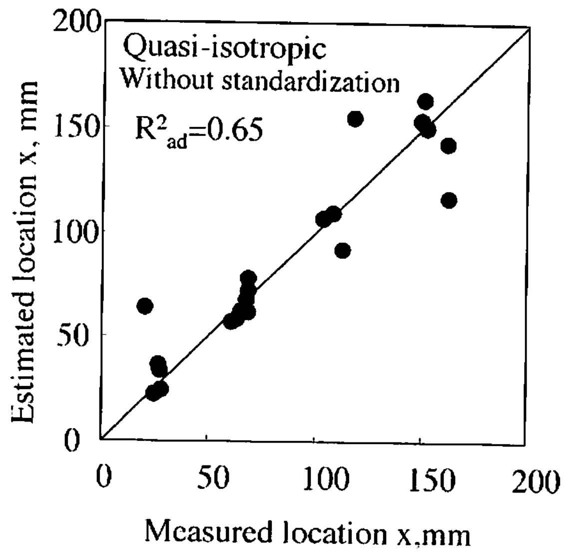
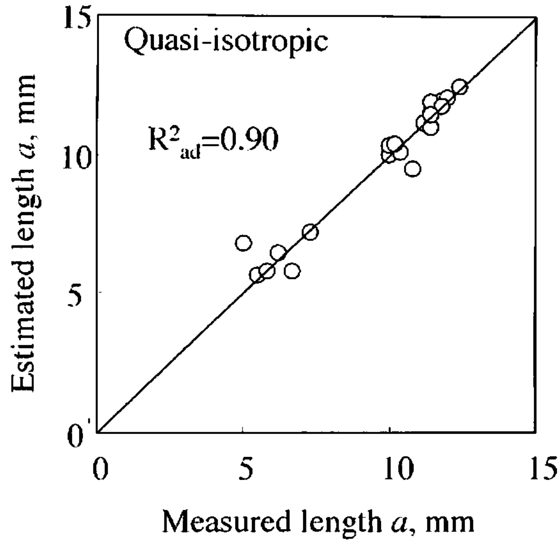


Figure 11 Estimated delamination location, for quasi-isotropic laminates



length. The value of R_{adj}^2 for each response surface was 0.76 for estimations of location and 0.87 for estimations of length.

Figure 10 shows delamination location for the quasi-isotropic specimen, and Figure 11 shows the delamination length. In Figure 10, the abscissa is the measured delamination location and the ordinate is the estimated location with the response surface without using standardizations of the measured electrical resistance changes. In Figure 11, the abscissa is the measured delamination length and the ordinate is the estimated length. The value of R_{adj}^2 for each response surface was 0.65 for estimations of location and 0.90 for length.

The standardization method of electrical resistance changes magnifies the electrical resistance change distributions. However, the method also magnifies experimental errors. In order to confirm the effect of the experimental error on the method, delamination identifications were performed in the present study with beam-type specimens made from cross-ply laminates and quasi-isotropic laminates. The effect of the stacking sequence on the estimation performance was also investigated. As a result, the electrical resistance change method with standardization was confirmed to be efficient even for the experimental results. Moreover, we have to make response surfaces for each individual stacking sequence, even when we use the standardization method.

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Figure 14 Standardized electrical resistance changes of two specimens. Cross-ply (triangle symbol) has a delamination of 5.2 mm length at a distance of 115 mm from the edge. Quasi-isotropic (Circle symbol) has a delamination of 5.0 mm at a distance of 113 mm from the edge

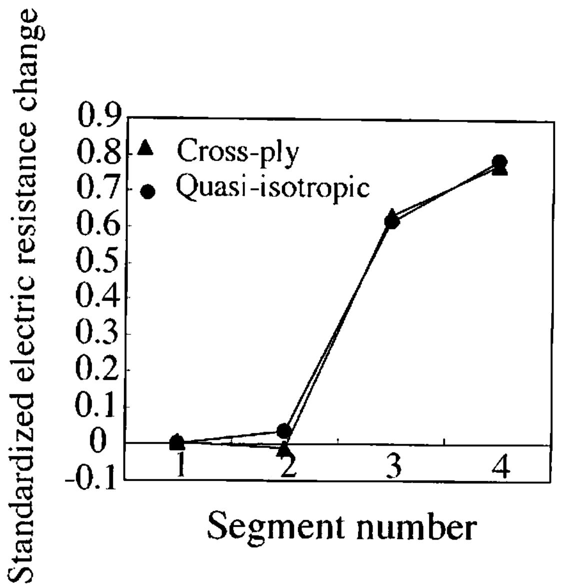


Figure 15 Standardized electrical resistance changes of two specimens. Cross-ply (triangle symbol) has a delamination of 11.2 mm length at a distance of 155 mm from the edge. Quasi-isotropic (Circle symbol) has a delamination of 11.4 mm at a distance of 153 mm from the edge

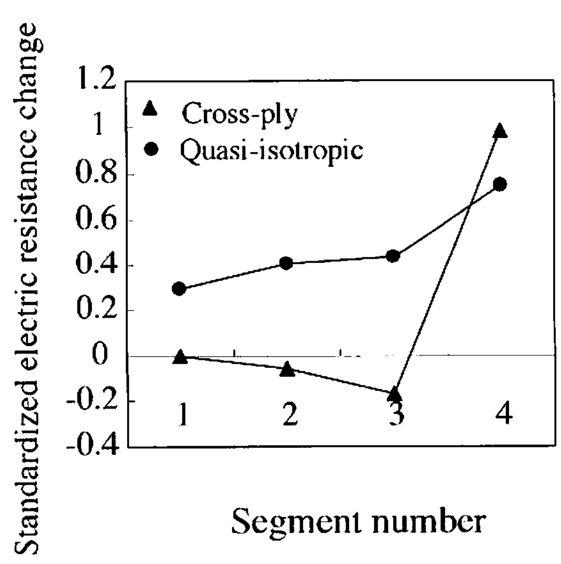
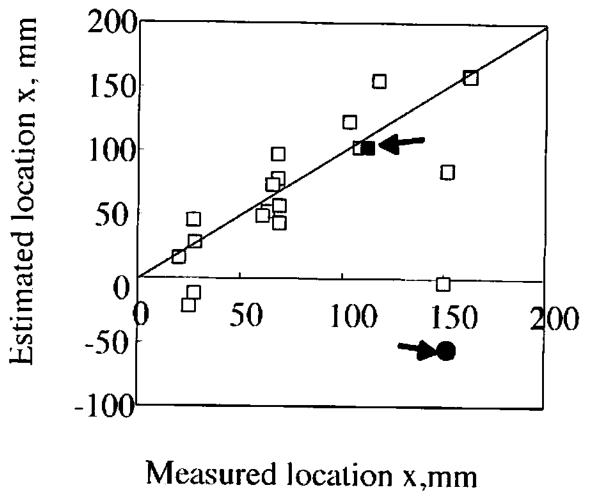


Figure 16 Estimated results of quasi-isotropic specimens using the response surface made from results of cross-ply specimens. The solid square symbol represents the result of Figure 14, and the solid circle symbol represents the result of Figure 15



specimens. The solid square symbol represents the estimation results of Figure 14 and the solid circle represents the result of Figure 15. Since most of the square symbols were near the diagonal line, this means that most of the estimated results agreed with the measured ones. Several results showed the large errors like the case of the solid circle symbol of Figure 15. All of these cases had different standardized electrical resistance changes from the similar case of the cross-ply specimen. So far the cause of this difference is not clear, but we can conclude that we have to obtain a response surface for each stacking sequence, even when we adopt the standardized electrical resistance changes.

CONCLUSIONS

The authors adopted an electrical resistance change method with standardization to identify delamination locations. Electrodes were mounted on composite laminates, and electrical resistance changes in each segment between the electrodes were measured by means of an electrical resistance bridge circuit. Using a large number of measured electrical resistance changes, the relationship between the electrical resistance changes and the delamination length was obtained with a response surface. The measured electrical resistance changes were standardized and used to obtain a relationship between the standardized electrical resistance changes and the delamination location.

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