Adv. Composite Mater., Vol. 12, No. 4, pp. 331–343 (2004)

Also available online - www.vsppub.com

Optimizations of stacking sequence and number of plies for laminated cylinders using GA with intron genes

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Received 16 April 2003; accepted 7 November 2003

Abstract—A new intron method is proposed to optimize stacking sequences and number of plies of composites. The intron method is developed from the recessive-gene-like repair method that is employed for implementation of stacking-sequence constraints without causing decrease of the design reliability of GA. The intron method is attempted for the optimizations of laminated cylinders to maximize the buckling load and minimize the number of plies. As a result, the intron method successfully obtained the real optimal stacking sequences and the number of plies without decrease of design reliabilities.

Keywords: Genetic algorithm; composites; optimization; intron; buckling.

1. INTRODUCTION

Stacking sequence optimizations are indispensable for designs of laminated composite structures. One stacking sequence optimization method is a graphical design method with lamination parameters proposed by Miki [1] and Fukunaga and Chou [2], which employs lamination parameters as continuous design variable and obtains an optimal stacking sequence of an angle ply laminate such as $\pm\theta$. For practical composite structures, however, available fiber angles are limited to a small set of 0°, ±45°, and 90° plies due to lack of experimental data and limitation caused by a hand lay-up fabrication process. These limitations give rise to stacking sequence optimizations that are combinatorial optimization problems with some constraints. Susuki [3] proposed a ranking method for strength optimizations of laminated composites for in-plane loadings. For general stacking sequence optimizations, genetic algorithms (GAs) are becoming popular [4–13] because GAs are generally admitted to be effective for combinatorial optimizations. For stacking

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sequence optimizations, however, there are several combinatorial constraints that are normally difficult to implement in GAs. For example, to prevent a tension-shear coupling effect, the number of angle plies ($\pm\theta°$-plies: $\theta \neq 0°, 90°$) must be balanced. To prevent a large matrix cracking effect, stacking of more than four plies of identical fiber angles must be avoided. Penalty methods to implement these combinatorial constraints reduce GA performance; for this, a recessive-gene-like repair strategy has been proposed by some authors [14, 15].

The recessive-gene-like repair deals stacking sequence optimizations of fixed number of plies. For actual aerospace structural applications of laminated composite materials, minimizations of number of plies are also significant problems to reduce total weight of target composite structures to satisfy design requirements. For example, let us consider the case of a maximization of buckling load of a target structure. The buckling load and the total weight increase with the increase of number of plies. Optimizations demand minimization of number of plies that satisfy a design requirement of buckling load and to maximize the buckling load of the fixed number of plies at the same time. This requires both minimization of number of plies and maximization of buckling load under the constraint of fixed number of plies by changing stacking sequences. To achieve this objective, an algorithm is needed that optimizes the number of plies and stacking sequences simultaneously.

An approach for simultaneous optimizations of both number of plies and stacking sequences has been proposed already by Harrison *et al.* [8], which employs an empty-gene method. This method adopts empty genes to represent vacant genetic loci in chromosomes: the vacant loci correspond to no existing ply. This empty gene method enables optimization of both number of plies and stacking sequences simultaneously. This method, however, does not consider the stacking sequence constraints, and the empty genes have to be moved to outer gene loci to represent actual stacking sequences, which causes additional complicated software coding.

In the present study, therefore, a new intron gene method is proposed for the optimizations of both number of plies and stacking sequence. The intron gene method is easily developed after slight modification to the previously developed recessive-gene-like repair, which has been installed to implement some constraints on stacking sequences in GA.

Genes are the unit components that provide genetic information. A sequence of genes makes chromosomes. For all living creatures, a chromosome has useless genes that are never decoded as genetic information. These useless genes are called intron. In the present study, the intron genes are utilized as an improved recessive-gene-like repair method. The intron is coded to correspond to empty genes as an additional recessive gene. In the previous study, the genes are expressed with trinary numbers. Here, the genes are expressed with quaternary numbers. The fourth number corresponds to the empty gene here: this simplifies program coding. This new intron method is applied to optimizations of both numbers of plies and stacking sequence to maximize buckling load of a composite cylinder of CFRP (carbon fiber reinforced plastic).

2. OPTIMIZATION PROBLEM

In the present study, a laminated composite cylinder is adopted as an example as shown in Fig. 1. Although the problem is simple, optimized results can be verified for this simple problem, and this problem includes both in-plane stiffness and out-of-plane stiffness. The method optimizes the stacking sequence using a minimum number of plies to satisfy design requirements while maximizing the buckling load. The new intron method, which is easily implemented using the recessive-gene-like repair method, is employed and other constraints on stacking sequences are also implemented here.

In Fig. 1, the longitudinal axis is the x-axis; the circumferential axis is ϕ; the radius axis is z. For this laminated composite cylinder, the radius is R, length is L and the thickness of the laminated wall of the cylinder is t. The thickness of each ply is h. Both ends of the cylinder are simply supported. In this analysis, $L = 3.0$ m, $R = 1.0$ m and $h = 0.125$ mm. The material used is T300/epoxy, the properties of which are $E_x = 133.44$ GPa, $E_y = 8.78$ GPa, $E_s = 3.254$ GPa and $\nu_{xy} = 0.26$.

Buckling load of axial compression of the simply supported orthotropic laminated composite cylinder ($\overline{N}_x/t$) is obtained with compliance matrix elements as follows.

(1) Axial-symmetric buckling $m = 1, n = 0$

$$\left(\frac{\overline{N}x}{t}\right)_s = \frac{2}{Rt}\sqrt{\frac{d_{11}}{a_{22}}}\left(\sqrt{1+\frac{b_{12}^2}{a_{22}d_{11}}}+\frac{b_{12}}{\sqrt{a_{22}d_{11}}}\right). \quad (1)$$

(2) Non-symmetric buckling $n \neq 0$

$$\left(\frac{\overline{N}x}{t}\right)_u = \frac{1}{Rt}\sqrt{\frac{d_{22}}{a_{11}}}\left(\Phi_1+\frac{(\Phi_3+\sqrt{\Phi_1\Phi_2+\Phi_3^2})^2}{\Phi_2}\right)\Bigg/\sqrt{\Phi_1\Phi_2+\Phi_3^2}, \quad (2)$$

where

$$\Phi_1 = \frac{a_{11}d_{11}}{a_{22}d_{22}}\mu^4 + 2\frac{d_{12}+2d_{66}}{\sqrt{d_{11}d_{22}}}\sqrt{\frac{a_{11}d_{11}}{a_{22}d_{22}}}\mu^2 + 1,$$

$$\Phi_2 = \mu^4 + 2\frac{a_{12}+0.5a_{66}}{\sqrt{a_{11}a_{22}}}\mu^2 + 1,$$

$$\Phi_3 = \frac{b_{12}}{a_{22}}\sqrt{\frac{a_{11}}{d_{22}}}\mu^4 + 2\frac{\{0.5\,(b_{11}+b_{22})-b_{66}\}}{\sqrt{a_{22}d_{22}}}\mu^2 + \frac{b_{21}}{\sqrt{a_{11}d_{22}}},$$

$$\mu^2 = \frac{\lambda^2}{n^2}\sqrt{\frac{a_{22}}{a_{11}}}, \quad \lambda = \frac{m\pi R}{L}.$$

In equations (1) and (2), a_{ij}, b_{ij} and d_{ij} are elements of elastic compliance matrix of a composite laminate [17]; m is the half-number of waves of buckling mode in the axial direction; n is the number of waves of buckling mode in the circumferential direction.

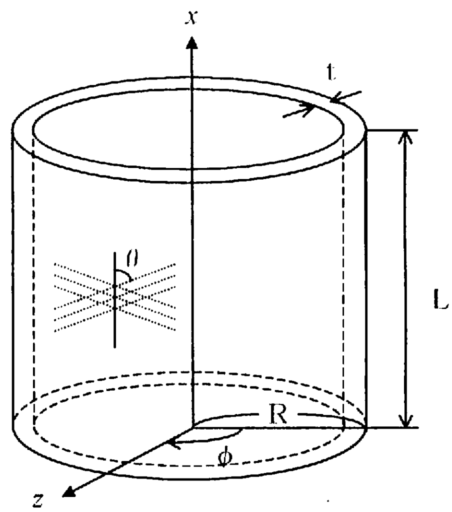

Figure 1. Configuration of laminated composite cylinder.

As a critical buckling load, the minimum-buckling stress between the axial-symmetric buckling and non-symmetric buckling is adopted here. For the analyses of the non-symmetric buckling, the minimum stress in equation (2) is sought by changing m and n from 1 to 20. When the minimum value of equation (2) is obtained at $m = 1$ and $n = 20$, calculation of buckling load of $m = 0$ is performed and these values are compared with each other. The smaller case is considered to be the critical bucking stress here. This is to reduce the searching time; but this method is only confirmed to be useful in this composite cylinder by means of a large number of analyses. This is not confirmed for another type of composite cylinder. The equations proposed by Tasi are limited to orthotropic laminates, and the laminates of the present study are not always orthotropic, although the available fiber angles of the symmetric laminates are limited to 0°, ±45° and 90°. The equations, therefore, are adopted as an approximate buckling load. Since the objective of the present study is to propose and investigate the effectiveness of the intron method, the buckling load analyses themselves are not exact.

3. IMPLEMENTATION OF GA AND INTRON METHOD

The genetic algorithm (GA) is one of the probabilistic optimization algorithms generated on the basis of the theory of evolution. The optimization process is a model of the law of the survival of the fittest of actual creatures: the fittest adaptable individual can leave offspring. This survival-of-the-fittest process is modeled in a computer program. Those individual with the highest fitness within the given environment are selected at high probability for reproductions of next generation, and the rest of the individuals in the group are curtailed. From the selected elitist group, the genetic information of the next generation is produced by means of crossovers and mutations.

In order to solve the optimization problems by means of this GA, design variables must be coded into a list of genes (chromosome), and a design example must

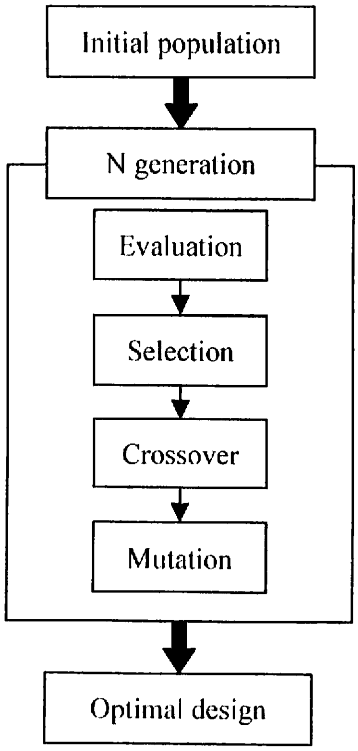

Figure 2. Process of genetic algorithm.

correspond to a chromosome or chromosomes that represent an individual. A group is made from these individuals, and the optimization is performed for the group using genetic procedures like fitness evaluations, selections, crossover and mutation. This process is illustrated in Fig. 2.

In the present study, available ply angles are limited to 0°, ±45° and 90° because few experimental data are available for practical design of composite laminates except for these ply angles and it is very difficult to make variable ply angles in a hand lay-up process. In our previous paper, these ply angles are expressed with trinary numbers: the numeral 0 corresponds to 0° ply; numeral 2 corresponds to 90° ply; odd occurrence of numeral 1 from the outermost ply corresponds to +45° and even occurrence of numeral 1 corresponds to −45° ply. Three constraints of stacking sequence are implemented using the recessive-gene-like repair method.

(1) Total number of plies of +45° is equal to the number of plies of −45°(balance rule).

(2) Difference of ply angles between adjacent plies must be equal to or smaller than 45° (adjacent ply angle rule).

(3) More than four contiguous plies of the same orientation is not allowed (4-contiguous ply rule).

The implementation of the recessive-gene like repair is explained later. Detailed explanation may be found in references [13] to [15].

Since the target structure is a symmetric laminate of N plies in the present study, the required length of chromosome (total number of genetic loci) is $N/2$. The symmetric laminate comprises 0°-plies, ±45°-plies and 90°-plies. Trinary numbers are applicable for representations of these fiber angles. Although the intron method uses quaternary numbers for representations of these fiber orientations and void plies, the repair system with trinary numbers is explained here first as an introduction.

Genes of each chromosome comprise trinary numbers. Trinary numbers comprise numerals 0, 1 and 2. Decoding of a trinary number to a stacking sequence is performed from the outermost ply because the outermost ply has a larger effect on the bending stiffness. Decoding from the outermost ply brings small repairs in the outer plies of the laminate that have large effect on the bending stiffness, and most repairs are automatically conducted in the inner plies.

Basic correspondences between trinary figures and fiber orientations are as follows: a gene of 0 corresponds to a 0°-ply; a gene of 1 corresponds to a 45°-ply or −45°-ply; a gene of 2 corresponds to a 90°-ply. For the gene of 1, odd occurrences of 1 from the outermost ply are decoded to 45°-plies and even occurrences of 1 are decoded to −45°-plies.

The constraints on numerals 1 and 2 are considered in the first stage of the repair, and the constraint on numeral 3 is considered in the second stage.

In the first stage, a gene is decoded from the outer genes on the basis of the basic correspondences listed above. At each decoding, the constraints on (1) and (2) are checked. When the decoded fiber orientation satisfies the constraints, the next gene is decoded. Otherwise, the decoding rule is shifted to the next fiber orientation.

For example, the correspondence of the violating gene of 0 is shifted to ±45° ply. In this case, a 45°-ply or −45°-ply is selected as the rule of genes of 1: the odd occurrence of 1 from the outermost ply is decoded to a 45°-ply and the even occurrence of 1 from the outermost ply is decoded to a −45°-ply. When the gene of 0 violates the rules even after the first shift of correspondence from 0°-ply to ±45° ply, the correspondence of the gene of 0 is shifted to 90°-ply again. When the gene of 0 still violates the rules after the second shift, the correspondence of the gene of 0 is fixed to 0°-ply to the basic correspondence. In a similar way, the correspondence of the gene of 1 is shifted from ±45°-ply to 90°-ply and 0°-ply when the basic decoding of the gene of 1 violates the rules. Similarly, after second shifts, when the decoding still violates the rule, the decoding of the gene of 1 is fixed to ±45°-ply. When a stacking sequence is violating the rules after the repair process, a penalty that is proportional to the number of violating plies is added to the fitness of the chromosome. This process is performed in all individuals. Almost all chromosomes satisfy the constraint rules after this repair process. Decoding example of the chromosome of {01122220} is shown in Fig. 3.

In the present study, the intron genes are also implemented to optimize number of plies. Genes are units of genetic information, and genes comprise DNA (deoxyribonucleic acid). Actual chromosomes have parts of genes that are not decoded as genetic information. These genes that are useless for reproductions are called intron.

The intron is implemented as an extension of the previously proposed recessive-gene-like repair method. The concept of the intron is added to the correspondences between the genes and the fiber angles. In our previous study, the genes are represented as trinary numbers, and each numeral has basic correspondence to the fiber angles. In the present study, quaternary numbers are adopted, and the numeral 3 represents the intron (*i*). The intron gene means no ply exists there. The basic correspondences between the numerals 0 to 2 and the fiber orientations are the same as the previous repair method. In this method, the numeral 3 is simply neglected here as the intron genes (see Fig. 4). This intron gene enables us to represent various numbers of plies with a fixed length of chromosomes. The recessive-gene-like repair adopted trinary numbers, and this intron method adopts quaternary numbers. The extension requires only slight changes in software coding. This change involves only two processes: change of numbers from trinary numbers

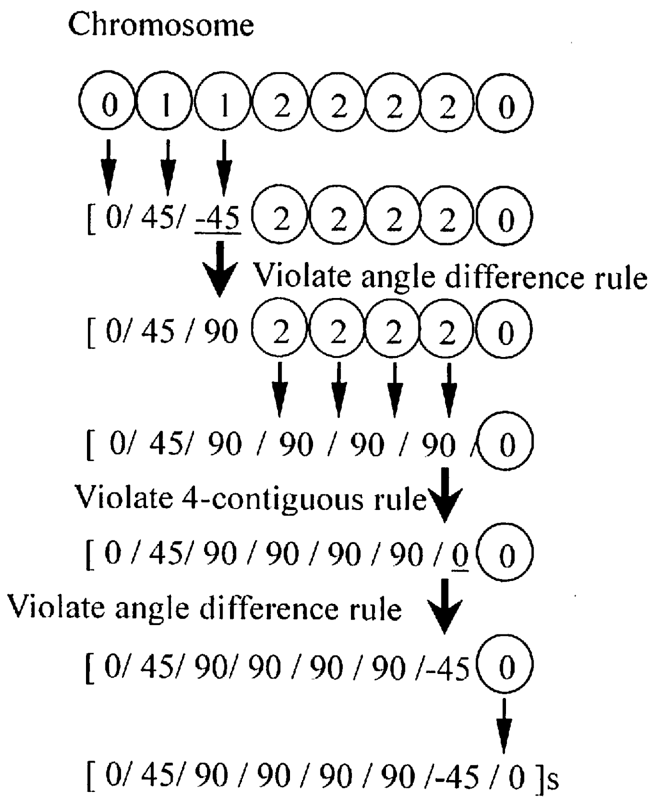

Figure 3. Example of decoding by the recessive-gene like repair method.

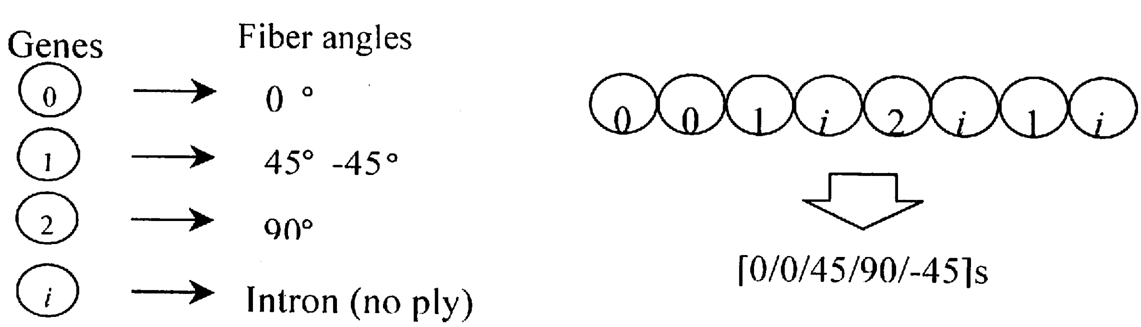

Figure 4. Correspondences of genes to actual fiber angles with the intron method.

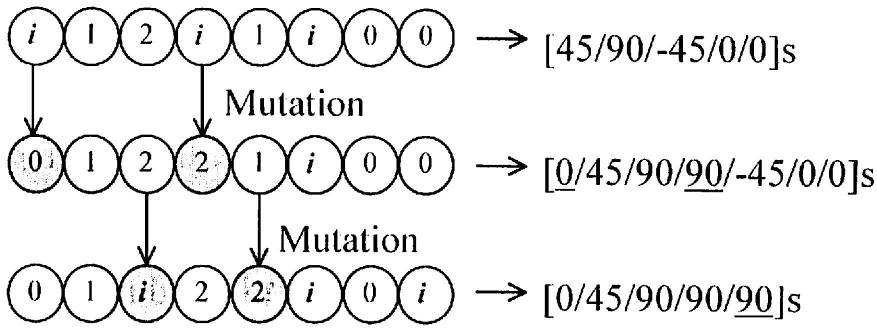

Figure 5. Decoding process using the intron method.

to quaternary numbers; and addition of a subprogram of decoding of the intron (simply neglect the intron).

This process is a simple extension of the recessive-gene like repair method. This method reserves order of genes. Direct changes of genes are prevented here as the same as the recessive genes. Example of decoding of the intron method is shown in Fig. 5.

In the present study, the optimization target is maximization of buckling stress of a composite cylinder, which means that the buckling stress is the fitness function of the GA here. The buckling stress given by the equations (1) and (2), however, is obtained on the basis of an orthotropic laminate. To prevent existence of strongly non-orthotropic laminates, orthotropic parameters of δ and γ are implemented here.

$$\gamma = \frac{D^*_{16}}{(D^{*3}_{11}D^*_{22})^{1/4}}, \quad \delta = \frac{D^*_{26}}{(D^*_{11}D^{*3}_{22})^{1/4}}. \tag{3}$$

When δ and γ are smaller than 0.2, the laminate can be approximated as an orthotropic laminate [18].

The objective function f is defined as follows for this optimization problem. In this problem, maximization of the fitness f is performed.

$$f = \left\{ p\left(16 - \frac{n}{2}\right) + q\frac{\min(\overline{N_X}/t)}{\sigma_d} \right\} \times 10 \Big/ (1 + \gamma + \delta), \tag{4}$$

$$\text{(1) when } \frac{\min(\overline{N_X}/t)}{\sigma_d} < 1, \quad (p, q) = (0, 1), \tag{5}$$

$$\text{(2) when } 1 \leqslant \frac{\min(\overline{N_X}/t)}{\sigma_d} < 2, \quad (p, q) = (1, 1), \tag{6}$$

$$\text{(3) when } \frac{\min(\overline{N_X}/t)}{\sigma_d} \geqslant 2, \quad (p, q) = (1, 0), \tag{7}$$

where σ_d is the required given design stress of buckling limit at which the cylinder does not buckle. The first term in the right side of equation (4) is to minimize the number of plies. The second term is to maximize the buckling load. Changing the values of p and q enables the change of weight of the two optimizations: minimization of number of plies and maximization of buckling stress in the given

Table 1.
GA parameters

Parameters	Values
Population size	10
Generation upper limit N_g	500
Probability of crossover (P_c)	0.8
Probability of mutation (P_m)	0.8
Number of mutated genes (per one chromosome)	2

number of plies. For example, when the buckling stress is smaller than the required stress (number of plies is underestimated), the first term is deleted to give a priority to maximization of the buckling stress. On the contrary, if the buckling stress is twice as large as the given required stress (number of plies is overestimated), the second term is deleted to give a priority to minimization of the number of plies. $N/2$ represents the half-number of plies. α and β are the penalty values when the laminate violates the orthotropic limitation given by the equation (3). The values of α and β are the γ and δ, respectively.

The parameters used in this GA are all shown in Table 1. In the present study, mutation is defined against each chromosome. The number of genes for mutations is fixed to 2 in a chromosome to prevent large changes by chance.

4. RESULTS AND DISCUSSION

In the present study, laminated CFRP cylinders that have a symmetric stacking sequence is considered. The total number of plies is an even number from 8 to 32. These laminates can be represented with chromosomes that have 16 genetic loci in a chromosome. In other words, a chromosome that has 16 genetic loci represents all laminates that are equal to or smaller than 32 plies with the intron method. To confirm the effectiveness of the intron method, firstly the optimal stacking sequences that maximize the buckling stress of each number of plies are obtained with the GA using the recessive-gene-like repair method of each fixed number of plies. The results are shown in Table 2. In this table, the maximum buckling load is also shown here.

Optimizations of both the number of plies and the maximizations of the buckling stress are performed under the given design requirement with the GA using the intron method. Three cases of buckling stress limits are considered: 50, 75 and 100 MPa. The results are shown in Table 3.

The obtained stacking sequences satisfy all of the constraints (1), (2) and (3) as shown in Table 3. The optimal result of the case that the given design requirement is 50 MPa is the same as the optimal stacking sequence of 16 plies in Table 2. Similarly, the optimal result of 75 MPa in Table 3 is the same as the optimal result of 22 plies in Table 2, and the optimal result of 100 MPa in Table 3 is the same

Table 2.
Optimal stacking sequence of each number of plies

Number of plies	Buckling stress (MPa)	Stacking sequence	Buckling mode
8	23.20	[0/45/90/−45]s	(1, ∞)
10	33.25	[90/45/0/−45/90]s [0/45/90/−45/90]s	(7, 5) (13, 20)
12	38.53	[0/45/90/−45/0/0]s [90/45/0/−45/90/90]s	(17, 20) (15, 14)
14	49.28	[90/45/0/−45/−45/90/45]s [0/45/90/−45/−45/0/45]s	(1, 0) (1, ∞)
16	58.52	[90/45/0/−45/0/45/90/−45]s [0/45/90/−45/90/45/0/−45]s	(5, 3) (11, 20)
18	67.11	[45/0/−45/90/45/90/−45/0/0]s [45/90/−45/0/45/0/−45/90/90]s	(1, 0) (1, ∞)
20	72.66	[45/0/−45/90/45/90/90/−45/0/0]s [45/90/−45/0/45/0/0/−45/90/90]s	(19, 20) (19, 20)
22	80.23	[0/45/90/−45/0/45/90/−45/−45/0/45]s [90/45/0/−45/90/45/0/−45/−45/90/45]s	(1, ∞) (1, 0)
24	91.10	[45/0/−45/90/90/45/0/−45/90/45/0/−45]s [45/90/−45/0/0/45/90/−45/0/45/90/−45]s	(1, ∞) (1, 0)
26	98.34	[45/0/−45/90/45/90/−45/90/45/90/−45/0/0]s [45/90/−45/0/45/0/−45/0/45/0/−45/90/90]s	(20, 19) (13, 15)
28	103.50	[45/0/−45/90/45/90/−45/0/45/90/90/−45/0/0]s [45/90/−45/0/45/0/−45/90/45/0/0/−45/90/90]s	(20, 19) (7, 8)
30	112.93	[0/45/90/−45/0/45/90/−45/90/45/0/−45/−45/0/45]s [90/45/0/−45/90/45/0/−45/0/45/90/−45/−45/90/45]s	(1, 0) (1, ∞)
32	121.72	[45/90/−45/0/0/45/90/−45/0/45/90/−45/90/45/0/−45]s [45/0/−45/90/90/45/0/−45/90/45/0/−45/0/45/90/−45]s	(14, 19) (16, 13)

Table 3.
Optimal stacking sequences of cylinders using the intron method

Design stress	Optimal stacking sequences	Buckling stress	Buckling mode
50 (MPa)	[90/45/0/−45/0/45/90/−45]s [0/45/90/−45/90/45/0/−45]s	58.52 (MPa)	(5, 3) (11, 20)
75 (MPa)	[0/45/90/−45/0/45/90/−45/−45/0/45]s [90/45/0/−45/90/45/0/−45/−45/90/45]s	80.23 (MPa)	(1, ∞) (1, 0)
100 (MPa)	[45/0/−45/90/45/90/−45/0/45/90/90/−45/0/0]s [45/90/−45/0/45/0/−45/90/45/0/0/−45/90/90]s	103.50 (MPa)	(20, 19) (7, 8)

as the optimal result of 28 plies in Table 2. These results show that the optimal stacking sequences obtained with the intron GA coincide with the optimal stacking sequences at each number of fixed plies with the previous recessive-gene-like repair method. Since the optimal results satisfy the buckling stress requirements, the GA with the intron method optimizes the number of plies with maximization of the buckling stress.

GA is one of the probabilistic searching methods. The goodness of fit of the algorithm must be confirmed on the basis of statistical evaluations. The intron method adopts quaternary numbers instead of using trinary numbers. This enlarges the entire design space. This enlargement may cause the reduction of design reliability (probability that GA gets the true optimal result). The design reliabilities of both methods are compared here: Design reliability of the previous recessive-gene like repair method of the fixed number of plies (true optimal number of plies) and design reliability of the intron method. The definition of the design reliability R is shown as follows.

$$R(\%) = \frac{N_0}{N_r} \times 100, \tag{8}$$

where N_r is the total number of runs and N_0 is the number of runs that obtained the true optimal stacking sequence. All runs of GA were performed with change in the seed of random numbers until the generation number amounts to the upper limit N_g ($N_g = 500$). The total number of runs for the cases that the number of plies is equal to or smaller than 20 is 100, and the cases that the number of plies is equal to or larger than 22 is 500. The design reliability is the probability of obtaining a true optimal result with a run of GA. The obtained reliability is shown in Table 4. Since the intron method optimizes the number of plies and maximization

Table 4.
Design reliability of the intron method

Number of plies	Reliability of recessive gene like repair (%)	Reliability of intron method (%)
8	100	—
10	100	—
12	100	—
14	100	—
16	100	100 (50 MPa)
18	100	—
20	100	—
22	83.8	89.8 (75 MPa)
24	36	—
26	98.8	—
28	85.6	74.6 (100 MPa)
30	10.2	—
32	37.2	—

of buckling stress without fixing the number of plies, the results of the intron method are listed at the optimal number of plies with the given buckling stress requirement in Table 4. For example, the optimization of the given stress of 50 MPa with the intron method has eight intron genes, except when the eight genes correspond to the actual fiber angles. The total number of genetic loci is 16 for the intron method. The result, however, is listed as 16-plies (8-gene case) in Table 4. As shown in Table 4, no reduction of the reliability is observed in the intron method. This indicates that the intron method is very useful for optimizations of number of plies and stacking sequences to maximize buckling stress.

5. CONCLUDING REMARKS

As a method to optimize both number of plies and stacking sequence of laminated composites, an intron method that can be obtained as an expansion of the previous recessive-gene-like repair method is proposed here. The applicability is confirmed with the optimizations of number of plies and stacking sequence to maximize the buckling stress of a composite cylinder. As a result, the intron method is easily implemented using the previous recessive-gene-like repair method, and it gives a similar design reliability.

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