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Design of experiments for stacking sequence optimizations with genetic algorithm using response surface approximation

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Abstract

This study describes a new method of experimental design to obtain a response surface of buckling load of laminated composites. Many evaluations for genetic algorithms for stacking sequence optimizations require high computational cost. That evaluation cost can be reduced by an approximation using a response surface. For a response surface for stacking sequence optimizations, lamination parameters are adopted as variables of the approximation function of the entire design space instead of ply angles for each ply. This study presents, proposes and investigates a new method of experimental design in detail. For most analytical tools, stacking sequences is demand as input data and lamination parameters cannot be applied directly to the tools. Therefore, the present study proposes and applies a new *D*-optimal set of laminates to the stacking sequence optimizations of the problem of maximization of buckling load of a composite cylinder. The new experimental design is a set of stacking sequences selected from candidate stacks using *D*-optimality. Consequently, the *D*-optimal set of laminates is shown to be effective for design of experiments of response surfaces for maximization of the buckling load of composite structures.
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Keywords: Design of experiments; Genetic algorithm; Optimum design; Composites; Response surface; Recessive gene; Buckling; Lamination parameter; Stacking sequence

1. Introduction

Laminated composite materials are usually fabricated from unidirectional plies of given thickness and with fiber orientations limited to a small set of angles, e.g., 0°, +45°, −45°, and 90°. The problem of designing such laminates for various strength and stiffness requirements is an integer-programming problem of selecting a required number of plies of each orientation and then determining an optimal stacking sequence. Although the branch and bound method has been used occasionally for solution (e.g., [1]), genetic algorithms are very popular for solving such stacking sequence optimization problems (e.g., [2–18]).

Unfortunately, integer programming solution techniques usually require large computational resources. For this reason, many composite structures optimization codes (e.g., PANDA2 [19]) use ply thickness as design variables with a fixed stacking sequence. Those codes perform continuous optimization. The solution of a continuous number is then rounded to an integer of number of plies. This approach yields sub-optimal designs; moreover, it cannot accommodate some constraints easily. An example of a constraint that is difficult to handle with the ply-thickness design variable is the limit of four on the number of contiguous plies of the same orientation used to reduce the chance of matrix cracking.

A previous work has suggested a two-stage approach to the design of composite laminates to overcome those difficulties [20]. In that paper, continuous design variables are used to obtain an initial solution first. Then an approximation to the structural response is constructed in the neighborhood of the continuous optimum. Finally a genetic algorithm is used to find the integer solution in that neighborhood. This approach is similar in its overall philosophy to a combination of the dual method with convex approximations.

A key ingredient in that previously proposed approach is the use of lamination parameters instead of the ply thickness as design variables for the continuous optimization problem and for approximation. Use of lamination parameters, introduced by Miki [21] and Fukunaga [22] for the solution of laminate design problems, gives two advantages over the use of ply

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thickness variables. First, they reduce the number of design variables considerably and allow easy use of approximation for changes in stacking sequence (e.g., Nagendra et al. [23]). Second, problems formulated with lamination parameters suffer much less from multiplicity of local optima, but have nearly the same performance. Although it is not essential for general application of this approach, the present work employs quadratic polynomial response surface approximations for both stages. The approximation is constructed for the entire design space in the first stage. In this respect, it is similar to the approach of Gangadharan et al. [24], who used thickness design variables. In the second stage the response surface is constructed in the neighborhood of the continuous optimum.

Our previous study [20] employed a response surface to reduce the evaluation cost of a genetic algorithm for stacking sequence optimizations. The variables of the response surface are the lamination parameters instead of the ply angles. The response surface is regressed from optimally selected design points using a *D*-optimal design. The use of lamination parameters reduces the number of analyses required for creation of the response surface. The accuracy of the response surface is enhanced by optimal selection of analysis points. A genetic optimization is adopted for obtaining stacking sequences of simply supported rectangular laminates to maximize buckling load. Thereby, we investigate the effectiveness of this approach. In the previous study, only out-of-plane lamination parameters are considered for optimization because of the simplicity of the optimization problem.

However, not only out-of-plane lamination parameters but also the in-plane lamination parameters have effects on the buckling load for some practical composite structures as in buckling load maximization of a cylindrical shell. These in-plane and out of-plane lamination parameters are not independent from each other. Therefore, to select a set of design points using the *D*-optimal without considering the hidden relationships between in-plane lamination parameters and out-of-plane lamination parameters may engender significant problems such as selection of an infeasible set of lamination parameters. Moreover, for most analytical tools, stacking sequences are the demanded input data and lamination parameters cannot be input directly into the analysis tool.

Therefore the present study proposes a new experimental design method for selection of practical laminates for creation of a response surface. The new method employs *D*-optimality for selections from a set of feasible laminates. The method is applied to a simple example of a stacking sequence optimization of buckling load maximization for a composite laminated cylinder. The buckling load of the cylinder is approximated using the response surface. This study uses a genetic algorithm with a recessive-gene-like strategy (see Ref. [12]) for stacking sequence optimizations.

2. Lamination parameters

In-plane stiffness terms of the symmetric laminates are represented with in-plane lamination parameters $\mathbf{V}_i^*$ $(i = 1, \ldots, 4)$ as

$$\begin{bmatrix} A_{11} \\ A_{22} \\ A_{12} \\ A_{66} \\ A_{16} \\ A_{26} \end{bmatrix} = h \begin{bmatrix} U_1 & V_1^* & V_2^* \\ U_1 & -V_1^* & V_2^* \\ U_4 & 0 & -V_2^* \\ U_5 & 0 & -V_2^* \\ 0 & \frac{1}{2}V_3^* & V_4^* \\ 0 & \frac{1}{2}V_3^* & -V_4^* \end{bmatrix} \begin{bmatrix} 1 \\ U_2 \\ U_3 \end{bmatrix}, \tag{1}$$

where h is the thickness of the laminate, U_i $(i = 1, \ldots, 5)$ are material invariants, and $\mathbf{V}_i^*$ represents in-plane lamination parameters. The material invariants are

$$\begin{aligned} U_1 &= \tfrac{1}{8}(3Q_{11} + 3Q_{22} + 2Q_{12} + 4Q_{66}), \\ U_2 &= \tfrac{1}{2}(Q_{11} - Q_{22}), \\ U_3 &= \tfrac{1}{8}(Q_{11} + Q_{22} - 2Q_{12} - 4Q_{66}), \\ U_4 &= \tfrac{1}{8}(Q_{11} + Q_{22} + 6Q_{12} - 4Q_{66}). \end{aligned} \tag{2}$$

Those equations use the following values

$$\begin{aligned} Q_{11} &= \frac{E_1}{1 - \nu_{12}\nu_{21}}, \quad Q_{22} = \frac{E_2}{1 - \nu_{12}\nu_{21}}, \\ Q_{12} &= \frac{\nu_{12}E_2}{1 - \nu_{12}\nu_{21}}, \quad Q_{66} = G_{12}. \end{aligned} \tag{3}$$

The in-plane lamination parameters are given as

$$\mathbf{V} = \begin{bmatrix} V_1^* \\ V_2^* \\ V_3^* \\ V_4^* \end{bmatrix} = \frac{2}{h} \int_0^{h/2} \begin{bmatrix} \cos 2\theta \\ \cos 4\theta \\ \sin 2\theta \\ \sin 4\theta \end{bmatrix} \mathrm{d}z, \tag{4}$$

where z is the coordinate of the thickness direction, the origin is located in the middle of the plate, and $\theta(z)$ is the fiber angle of the location of z.

Out-of-plane stiffness terms of the laminates are represented with out-of-plane lamination parameters $\mathbf{W}_i^*$ as

$$\begin{bmatrix} D_{11} \\ D_{22} \\ D_{12} \\ D_{66} \\ D_{16} \\ D_{26} \end{bmatrix} = \frac{h^3}{12} \begin{bmatrix} U_1 & W_1^* & W_2^* \\ U_1 & -W_1^* & W_2^* \\ U_4 & 0 & -W_2^* \\ U_5 & 0 & -W_2^* \\ 0 & \frac{1}{2}W_3^* & W_4^* \\ 0 & \frac{1}{2}W_3^* & -W_4^* \end{bmatrix} \begin{bmatrix} 1 \\ U_2 \\ U_3 \end{bmatrix}. \tag{5}$$

The out-of-plane lamination parameters are defined as

$$\mathbf{W} = \begin{bmatrix} W_1^* \\ W_2^* \\ W_3^* \\ W_4^* \end{bmatrix} = \frac{24}{h^3} \int_0^{h/2} z^2 \begin{bmatrix} \cos 2\theta(z) \\ \cos 4\theta(z) \\ \sin 2\theta(z) \\ \sin 4\theta(z) \end{bmatrix} \mathrm{d}z. \tag{6}$$

3. Response surface method

3.1. Curve fitting

Response surface methodology is applied to obtain an approximation to a response function in terms of predictor variables. The response model is generally written as

$$y = F(x_1, x_2, \ldots, x_n) + \varepsilon, \tag{7}$$

where y is the response, x_i $(i = 1, \ldots, n)$ are predictor variables, and ε is an error term. If F is a model that exactly describes the physical process being modeled, ε may be considered to represent random error resulting from numerical or experimental noise. The function F is normally selected to be a polynomial. For a quadratic polynomial, F is written as

$$y = \beta_0 + \sum_{i=1}^{n} \beta_i x_i + \sum_{i=1, j>i}^{n} \beta_{ij} x_i x_j, \tag{8}$$

where β represents unknown coefficients. Let us consider a case employing two variables and a quadratic polynomial. The response surface is expressed as

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1^2 + \beta_4 x_2^2 + \beta_5 x_1 x_2. \tag{9}$$

For ease of notation, let $x_3 = x_1^2$, $x_4 = x_2^2$ and $x_5 = x_1 x_2$. Thereby, the equation becomes

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5. \tag{10}$$

The unknown coefficients β_i $(i = 0, \ldots, 5)$ in Eq. (10) are estimated by a linear multiple regression. The linear multiple regression model is rewritten in matrix form as

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}, \tag{11}$$

where

$$\mathbf{Y} = \begin{Bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{Bmatrix}, \quad \mathbf{X} = \begin{bmatrix} 1 & x_{11} & x_{12} & \cdots & x_{1k} \\ 1 & x_{21} & x_{22} & \cdots & x_{2k} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_{n1} & x_{n2} & \cdots & x_{nh} \end{bmatrix},$$
$$\boldsymbol{\beta} = \begin{Bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_k \end{Bmatrix}, \quad \text{and} \quad \boldsymbol{\varepsilon} = \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{Bmatrix}, \tag{12}$$

where $\boldsymbol{\varepsilon}$ is an error vector.

The unbiased estimator $\mathbf{b}$ of the coefficient vector $\boldsymbol{\beta}$ is obtained using the least square error method as

$$\mathbf{b} = (\mathbf{X}^{\mathrm{T}}\mathbf{X})^{-1}\mathbf{X}^{\mathrm{T}}\mathbf{Y}. \tag{13}$$

The variance–covariance matrix of $\mathbf{b}$ is obtained as

$$\mathrm{cov}(b_i, b_j) = C_{ij} = \sigma^2(\mathbf{X}^{\mathrm{T}}\mathbf{X})^{-1}, \tag{14}$$

where σ is the error of $\mathbf{Y}$. The estimated value of σ is

$$\sigma^2 = \frac{SS_{\mathrm{E}}}{n - k - 1}. \tag{15}$$

In that equation, SS_{E} is the squared sum of errors expressed as the following:

$$SS_{\mathrm{E}} = \mathbf{Y}^{\mathrm{T}}\mathbf{Y} - \mathbf{b}^{\mathrm{T}}\mathbf{X}^{\mathrm{T}}\mathbf{Y}. \tag{16}$$

The adjusted coefficient of multiple determination R^2_{adj} (R-square-adjusted) is used to evaluate the performance of the approximation of the response surface

$$R^2_{\mathrm{adj}} = 1 - \frac{SS_{\mathrm{E}}/(n - k - 1)}{S_{yy}/(n - 1)}. \tag{17}$$

In that calculation, S_{yy} represents the sum of squares. It is calculated as the following:

$$S_{yy} = \mathbf{Y}^{\mathrm{T}}\mathbf{Y} - \frac{\left(\sum_{i=1}^{n} y_i\right)^2}{n}. \tag{18}$$

Each coefficient of the response surface can be tested using the t-statistic. The t-statistic of the coefficient b_j is

$$t_0 = \frac{b_j}{\sqrt{\sigma^2 C_{jj}}}, \tag{19}$$

where C_{jj} is the element of number jj of the variance–covariance matrix of Eq. (14).

3.2. Design of experiments

A set of data points where the response "y" is calculated or measured is required to obtain the response surface. It is desirable to select a better set of data points that maximizes accuracy of the approximation for a given number of data points. The process of selecting a set of the better data points is known as design of experiments (DOE). Standard DOE arrangements are available for box-like or spherical domains. However, we cannot use such simple geometrical arrangement of points for more complicated design domains. Instead, a D-optimal computer-generated DOE is used to select data points. The D-optimality criterion which minimize the sensitivity of $\mathbf{b}$ to errors in y are the most widely used criterion for selection of data points for computer generated DOE.

The D-optimality criterion maximizes the determinant of the moment matrix, $\mathbf{M}$, which is defined (e.g., Myers and Montgomery [25]) as

$$\mathbf{M} = \frac{\mathbf{X}^{\mathrm{T}}\mathbf{X}}{k}. \tag{20}$$

A related measure is the D-efficiency

$$D_{\mathrm{eff}} = \frac{\left(\mathrm{Det}[\mathbf{X}^{\mathrm{T}}\mathbf{X}]\right)^{1/p}}{k}, \tag{21}$$

where p is the total number of parameters included in the response surface model (the order of the matrix $\mathbf{M}$). If all variables are normalized so that they vary from −1 to 1, then the maximum value of the D_{eff} is 1. Furthermore, the quality of the set of points can then be measured by D_{eff}.

Our previous paper [20] presented a stacking sequence optimization using a genetic algorithm (GA). The response surface method was employed to reduce the computational cost of evaluations of laminates for the GA. The number of variables was reduced by use of the lamination parameters as variables of the response surface. The typical response surface is represented as

$$y = \lambda = \beta_0 + \beta_1 W_1^* + \beta_2 W_2^* + \beta_3 W_1^{*2} + \beta_4 W_2^{*2} + \beta_5 W_1^* W_2^*, \qquad (22)$$

where y is the response, λ is the buckling load, and β_i are coefficients obtained by regression.

This method is applied to the problem to maximize buckling load of a simply supported rectangular plate with the GA. A D-optimal design of experiments is performed in this study to create the response surface. From candidate points generated by equally spaced points in the out-of-plane lamination parameters (W_1^*, W_2^*), 12 appropriate points were selected with D-optimality in the previous paper [20].

4. Stacking sequence optimization using GA

4.1. Coding and repair

An elitist genetic algorithm (GA) is adopted to solve the combinatorial problem of stacking sequence optimization of a composite plate coded by one chromosome. Only half of the plies of a laminate are coded in the chromosome because of symmetry.

Trinary numbers are employed here for coding of ply angles as genes: each gene has a value of 0, 1 or 2. Basically the number 0 corresponds to the 0° ply and the number 2 corresponds to the 90° ply. 'Odd' occurrences, i.e., the first (outermost), third, fifth, etc. of number 1 correspond to 45° plies, whereas 'even' occurrences correspond to −45° plies. For example, a chromosome of [0/1/1/2/1/2/0] is decoded into a stacking sequence of $[0/45/{-45}/90/45/90/0]_s$. There is one unbalanced 45° ply when the number of occurrences of 1 is odd. This is repaired by the repair system described below.

4.2. Repair system

Decoding starts with the outermost ply: if five contiguous plies of the same orientation are encountered, the innermost gene value is incremented by one before translation. Note that the gene is not changed in the chromosome, but it is translated as if its value were one higher. Additional details including treatment of plies near the plane of symmetry may be found in Ref. [12].

Even if the laminated plate is unbalanced, there is only a single unbalanced 45° ply. To repair this unbalance, the repair system first attempts to replace one +45°-ply with a 90°-ply or a 0°-ply. The 45°-ply position replaced by a 90°-ply or a 0°-ply is the innermost 45°-ply that can be replaced without violating the four-contiguous-ply rule: the same fiber angle plies must not stacked more than four plies. If it is impossible to effect this repair, the innermost 90°-ply or 0°-ply is replaced by a 45°-ply. For details see Ref. [12].

4.3. Genetic operators

Parents are selected by the roulette wheel method using a two-point crossover with probability $P_c = 0.8$. Mutation is applied to two genes; the probability of mutation per gene is chosen so that the probability of mutation in a chromosome is 80%. Following Le Riche and Haftka [3], a permutation operator that interchanges the position of two genes is applied with probability of 100%. Parameters of the GA are identical to those in Ref. [20].

4.4. GA with response surface

The most time consuming process for the GA of structural optimizations is the iterative evaluations because the GA optimization procedure requires multiple evaluations of the fitness of each chromosome. Optimizing a stacking sequence of a laminated structure to maximize the buckling load usually requires computationally high-cost, repetitive FEM analysis. Therefore, a previous study employed a response surface that provides approximate buckling load [20]. The response surface variables were out-of-plane lamination parameters instead of respective fiber angles. This reduced nonlinearity of the problem while reducing the number of variables for thick laminates. The previous paper applied the method to the stacking sequence optimization to maximize the buckling load of a simply supported rectangular plate. Only out-of-plane lamination parameters are considered for that problem; only 12 calculations were needed to create the response surface.

5. New design of experiments for composite laminates

Our previous paper [20] proposes the optimization problem of maximizing the buckling load of a simply supported rectangular laminated plate. The problem requires only out-of-plane lamination parameters (W_1^*, W_2^*) because the problem is dominated only by bending stiffness. Twelve laminates are selected using the D-optimal from the equal spacing 2000 candidates in out-of-plane lamination parameters (W_1^*, W_2^*) to create a response surface of estimations of the buckling load of Eq. (22).

Simple equal spacing candidates of laminates are not always feasible laminates because the in-plane lamination parameters (V_1^*, V_2^*) and out-of-plane lamination parameters (W_1^*, W_2^*) are not independent of each other. Therefore this method cannot be applied to this problem which requires both in-plane and out-of-plane lamination parameters for creation of an approximation function of the objective function. In the present study, a feasible set of laminates is selected from among all feasible laminates. Let us consider the case of a 16-ply laminate. The total number of entire feasible laminates is $3^8 = 6561$ because we consider only a symmetric laminate and we adopt trinary numbers for genes that represent the fiber angles. We can select feasible laminates from the set of feasible laminates using the *D*-optimal.

The practical procedure is as follows. First, decimal numbers from 0 to 6560 are transformed to corresponding trinary numbers. For example, the decimal number 128 is transformed to "00011202". The trinary numbers are decoded to stacking sequences in consideration of the balance rule with the recessive gene like repair strategy. For example, the "00011202" is decoded to the stacking sequence of $[0/0/0/45/-45/90/0/90]_s$. The in-plane lamination parameters (V_1^*, V_2^*) and the out of-plane lamination parameters (W_1^*, W_2^*) are calculated for all feasible laminates. The entire set of feasible laminates is plotted in in-plane lamination parameter coordinates and out-of-plane lamination parameter coordinates as shown in Fig. 1(a) and (b), respectively. Dots in the figures represent feasible laminates. Similar laminates are all deleted from candidates for simplicity of calculation of the *D*-optimality because the set of the all laminates has too many similar coordinates especially in the out-of-plane lamination parameter coordinates. The set of lamination parameters of a laminate is regarded as a vector of four-dimensional coordinates. All similar laminates located within a sphere that has a radius of 20% of the length of the vector are deleted from the set of candidates. Appropriate laminates are selected by *D*-optimality from the remaining laminates.

In the present study, a quadratic polynomial is employed as a function of a response surface. Fifteen unknown coefficients exist in the quadratic polynomial function of four variables because the number of lamination parameters is four. The number of experiments in the present study is determined to be 36 because the

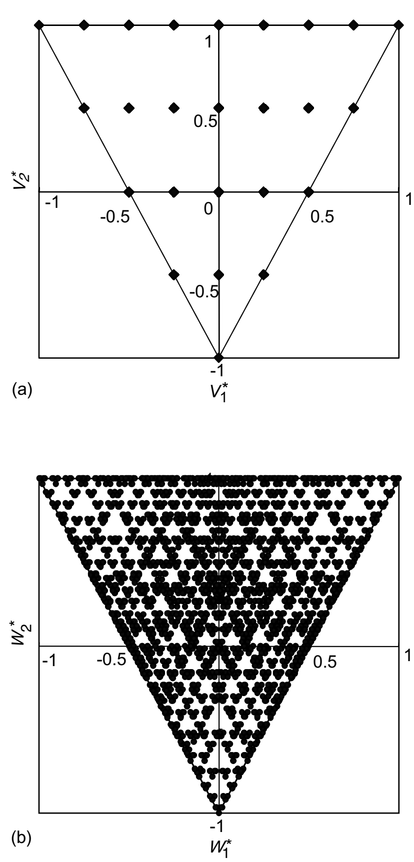


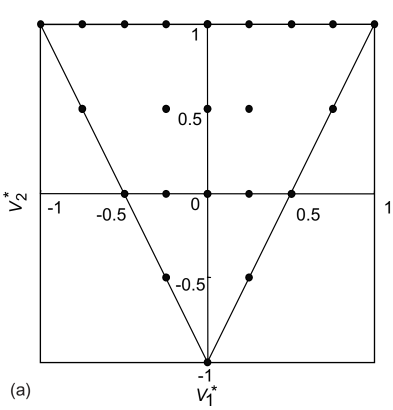
Fig. 1. Entire stacking sequences of symmetric laminates of 16 piles: (a) in-plane lamination parameters, and (b) out-of-plane lamination parameters.

Table 1
Selected *D*-optimal set of laminates

1	$[0/0/0/0/0/0/0/0]_s$
2	$[0/0/0/0/0/0/45/-45]_s$
3	$[0/0/0/0/0/90/90/90]_s$
4	$[0/0/0/0/45/-45/45/-45]_s$
5	$[0/0/0/0/90/90/90/90]_s$
6	$[0/0/45/-45/45/-45/45/-45]_s$
7	$[0/0/45/-45/45/-45/90/90]_s$
8	$[0/0/90/90/45/-45/45/-45]_s$
9	$[0/0/90/90/90/90/90/90]_s$
10	$[0/45/-45/45/-45/90/90/90]_s$
11	$[0/90/0/90/90/90/0/0]_s$
12	$[0/90/90/90/90/90/45/-45]_s$
13	$[0/90/90/90/90/90/90/90]_s$
14	$[45/0/0/-45/0/45/0/-45]_s$
15	$[45/-45/0/0/0/0/0/0]_s$
16	$[45/-45/0/0/0/0/90/90]_s$
17	$[45/-45/0/0/0/90/90/90]_s$
18	$[45/-45/45/-45/0/0/0/0]_s$
19	$[45/-45/45/-45/45/-45/0/0]_s$
20	$[45/-45/45/-45/45/-45/45/-45]_s$
21	$[45/-45/45/-45/45/-45/90/90]_s$
22	$[45/-45/45/-45/90/90/90/90]_s$
23	$[45/-45/90/90/90/0/0/0]_s$
24	$[45/-45/90/90/90/90/90/90]_s$
25	$[90/0/0/0/0/0/0/0]_s$
26	$[90/45/-45/45/-45/0/0/0]_s$
27	$[90/45/-45/90/45/90/-45/90]_s$
28	$[90/90/0/0/0/0/0/0]_s$
29	$[90/90/0/0/0/0/45/-45$
30	$[90/90/45/-45/45/-45/0/0]_s$
31	$[90/90/45/-45/45/-45/45/-45]_s$
32	$[90/90/90/90/0/0/0/0]_s$
33	$[90/90/90/90/45/-45/45/-45]_s$
34	$[90/90/90/90/90/90/0/0]_s$
35	$[90/90/90/90/90/90/45/-45]_s$
36	$[90/90/90/90/90/90/90/90]_s$

number of experiments required for regression is approximately more than twice the number of coefficients. The *D*-optimal design was performed using JMP software of SAS. The selected laminates are shown in Table 1 and plots of all the selected laminates are shown in Fig. 2(a) and (b). The value of D_{eff} of the set is 6.3%, which is lower than $D_{\text{eff}} = 25.6\%$ of the previous paper [20]. In that paper, the *D*-optimal set of points was selected from equally distributed points. The new *D*-optimal set of the laminates is only selected from feasible laminates. The low D_{eff} of the *D*-optimal set of the laminates shows that a severe constraint exists between the in-plane lamination parameters and the out-of-plane lamination parameters for feasible laminates.

The selected *D*-optimal laminates are not limited to 16-ply laminates. For example, let us consider the case whereby the number of plies is $2N$ (N is positive integer) for a laminate. Let the normal thickness of a ply be h_p. When we regard the laminate comprising plies of the thickness $h^* = h_pN/8$, the *D*-optimal laminates shown in Table 1 have identical lamination parameters as those shown in Eq. (9). That means the set is *D*-optimal. Of course, the *D*-optimal set is different for the case of a different number of plies. That difference, however, can be neglected. For example, for the case of the number of plies is 12, D_{eff} is 6.2%, and D_{eff} is 6.4% for the case where the number of plies is 20. Increasing the number of plies simply increases the density of the plots in Fig. 1; the increase of the number of plies does not mean a change of the feasible design space of the lamination parameters. This causes the small changes of the D_{eff} with the increase of the number of plies. We can use the *D*-optimal set of laminates in Table 1 for any laminate by changing the thickness of plies to h^* because the D_{eff} is approximately constant for all laminates.



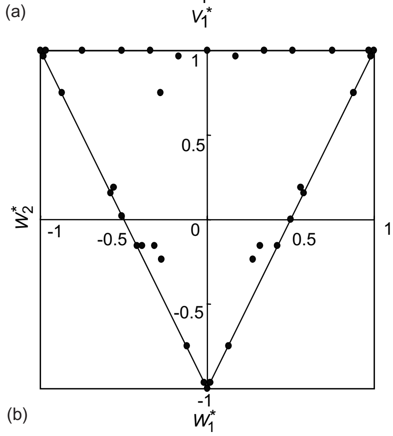


Fig. 2. Selected stacking sequences of *D*-optimal set of laminates: (a) in-plane lamination parameters, and (b) out-of plane lamination parameters.

6. Application of the new method

6.1. Optimization problem

A stacking sequence optimization to maximize buckling loads of a composite cylinder under axial compression loading is conducted here to confirm effectiveness of the new *D*-optimal laminates. Fig. 3 shows the configuration of the composite cylinder. The outer radius is R and the length of the cylinder is L. Thickness of the composite laminate is t. For s composite cylinder composed of thin orthotropic laminates, the analytical buckling load has already been obtained by Tasi [26] as follows:

(i) Axial symmetric buckling ($m = 0, n = 1$):

$$\left(\frac{\overline{N}_X}{t}\right)_s = \frac{2}{Rt}\sqrt{\frac{d_{11}}{a_{22}}}\left(\sqrt{1+\frac{b_{12}^2}{a_{22}d_{11}}}+\frac{b_{12}}{\sqrt{a_{22}d_{11}}}\right). \tag{23}$$

(ii) Non-axial-symmetric buckling ($n \neq 0$):

$$\left(\frac{\overline{N}_X}{t}\right)_u = \frac{1}{Rt}\sqrt{\frac{d_{22}}{a_{11}}}\left(\Phi_1+\frac{\left(\Phi_3+\sqrt{\Phi_1\Phi_2+\Phi_3^2}\right)^2}{\Phi_2}\right)\Bigg/\sqrt{\Phi_1\Phi_2+\Phi_3^2}, \tag{24}$$

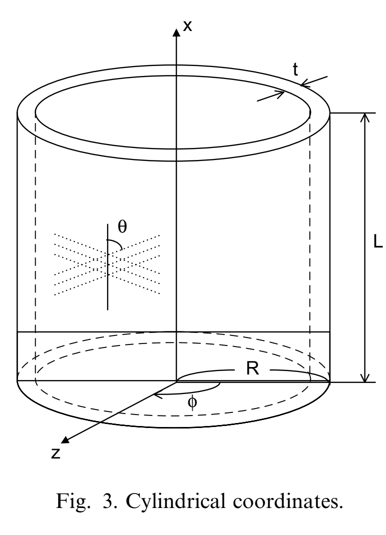


Fig. 3. Cylindrical coordinates.

where the following are true

$$\Phi_1 = \frac{a_{11}d_{11}}{a_{22}d_{22}}\mu^4 + 2\frac{d_{12}+2d_{66}}{\sqrt{d_{11}d_{22}}}\sqrt{\frac{a_{11}d_{11}}{a_{22}d_{22}}}\mu^2 + 1, \tag{25}$$

$$\Phi_2 = \mu^4 + 2\frac{a_{12}+0.5a_{66}}{\sqrt{a_{11}a_{22}}}\mu^2 + 1, \tag{26}$$

$$\Phi_3 = \frac{b_{12}}{a_{22}}\sqrt{\frac{a_{11}}{d_{22}}}\mu^4 + 2\frac{\{0.5(b_{11}+b_{22})-b_{66}\}}{\sqrt{a_{22}d_{22}}}\mu^2 + \frac{b_{21}}{\sqrt{a_{11}d_{22}}}, \tag{27}$$

$$\mu^2 = \frac{\lambda^2}{n^2}\sqrt{\frac{a_{22}}{a_{11}}}, \tag{28}$$

$$\lambda = \frac{m\pi R}{L}. \tag{29}$$

In Eqs. (23)–(29), a_{ij}, b_{ij}, d_{ij} $(i,j = 1,2,6)$ are elements of the compliance matrix of composite laminates. The m is the half-wave number of buckling mode in the axial direction, whereas n is the circumferential wave number of buckling mode. Only symmetric laminates are considered in the present study. All elements of b_{ij} vanish in the equations above. Eqs. (23) and (24) give an exact buckling load only for a case where each laminate has orthotropic stiffness. However, the equations are adopted in the present study as an approximate analysis for buckling of thin cylinders.

Eq. (23) gives the axial symmetric buckling load; Eq. (24) gives the non-axial-symmetric buckling loads for all combinations of m, n ($m = 1,\dots,20$, $n = 1,\dots,20$). In these buckling loads, the smallest buckling load is selected as the critical buckling load of the composite cylinder. In the case where n of the critical buckling load is 20, the buckling load of the case of $n = \infty$ is additionally calculated. If the buckling load of the case of $n = \infty$ is smaller than that of the case of $n = 20$, the buckling load of the case of $n = \infty$ is decided selected as the approximated buckling load.

The problem solved in the present paper is the design of an optimal stacking sequence that provides the maximum buckling load for a composite cylinder. The objective function of GA is selected as

$$f = \min(\overline{N}_X/t)/(1+\alpha+\beta). \tag{30}$$

The buckling load of Eqs. (23) and (24) is not accurate for laminates with large values of bending and twisting coupling stiffness D_{16} and D_{26}. The α and β in Eq. (30) are penalties for laminates that have large values of D_{16}

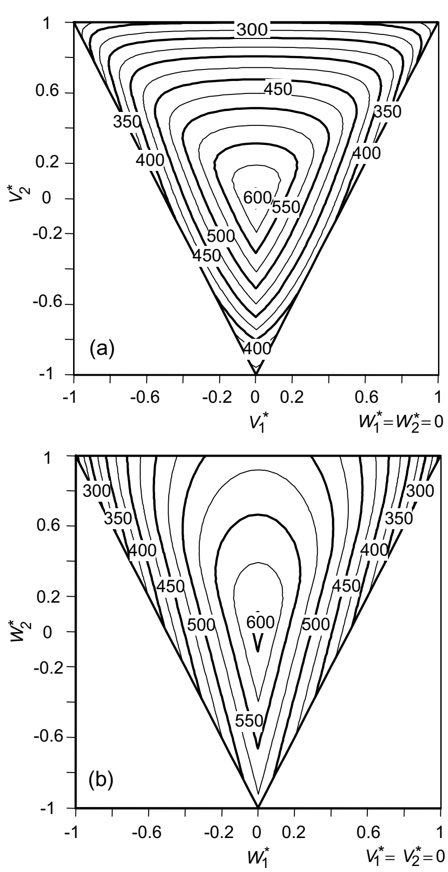


Fig. 4. Contour plots of buckling load of composite cylinder: (a) in-plane, and (b) out-of-plane.

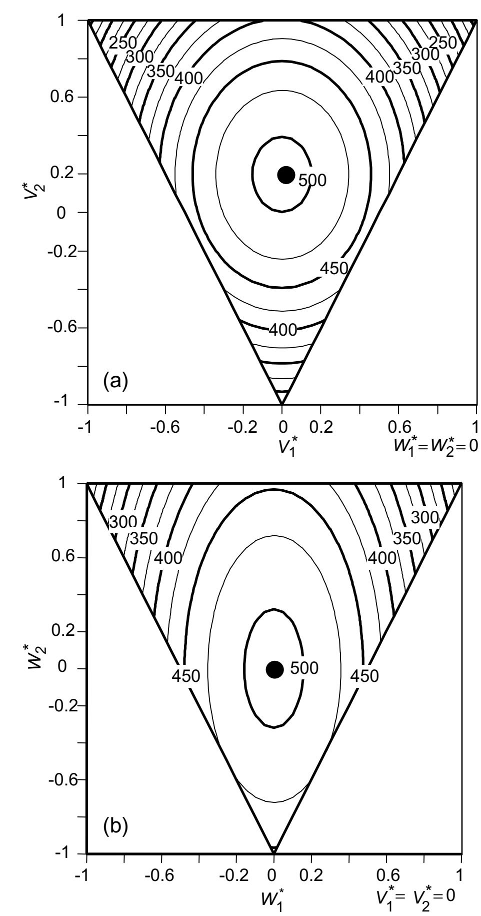


Fig. 5. Contour plot of response surface: (a) in-palne, and (b) out-of-plane.

Table 2
Optimal stacking sequences obtained by GA with response surface

	Optimal stacking sequences	V_1^*	V_2^*	W_1^*	W_2^*	Reliability (%)
Obtained from RS	$[45/0/90/-45/90/45/45/0]_s$ $[45/90/0/-45/0/45/-45/90]_s$	0	0	0	0	100

and D_{26}. The magnitude of the D_{16} and D_{26} terms can be measured by the following two non-dimensional terms:

$$\gamma = \frac{D_{16}}{(D_{11}^3 D_{22})^{1/4}}, \quad \delta = \frac{D_{26}}{(D_{22}^3 D_{11})^{1/4}}. \tag{31}$$

Nemeth [27] has shown that the effect of D_{16} and D_{26} can be neglected when both γ and δ are lower than 0.2. In a case where γ and δ are higher than 0.2, the values of the γ and δ are directly set to penalties α and β in the Eq. (30), respectively.

6.2. Results and discussion

It is possible to obtain the exact optimal laminate by solving numerous points of lamination parameters on the basis of the assumption that lamination parameters are completely independent of each other and that lamination parameters are continuous variables. The optimal point obtained with the assumption is $(V_1^*, V_2^*, W_1^*, W_2^*) = (0, 0, 0, 0)$. The optimal point with the assumption does not always mean that is a feasible point for practical laminates because there are constraints for practical laminates. The search for an optimal stacking sequence is a combinatorial optimization problem: the GA is required to solve it.

The practical design space near the optimal point is shown in Fig. 4. Fig. 4(a) shows the contour plot of the buckling load of the case that the out-of-plane lamination parameters are fixed to $(W_1^*, W_2^*) = (0, 0)$. In the same way, Fig. 4(b) shows the contour plot of the buckling load of the case where in-plane lamination parameters are fixed to $(V_1^*, V_2^*) = (0, 0)$.

A response surface to estimate the buckling load is created from analytical results for 36 laminates shown in Table 1. A quadratic polynomial is employed for the response surface function. All coefficients are tested using t-statistics (see Ref. [25]); the coefficients that are not effective for regression are deleted one by one. The obtained best response surface is the following:

$$y = \frac{\overline{N_x}}{t} = 449.87 + 63.87V_2^* - 266.73V_2^{*2} + 340.32W_1^{*2} - 161.25V_1^* V_2^* - 246.55V_2^* W_1^* - 59.99W_1^* W_2^*. \tag{32}$$

The adjusted coefficient of multiple determination R_{adj}^2 is 0.904 and the residual sum of squares is 31.95. The high R_{adj}^2 represents that the response surface has good fit to the buckling load. The contour plot of the buckling load near the optimum is shown in Fig. 5. The constraint of Fig. 5 is the same as that of Fig. 4. The optimal point of the response surface is $(V_1^*, V_2^*, W_1^*, W_2^*) = (0, 0.198, 0, 0)$ when we consider that the lamination parameters are independent with each other and continuous. The optimal point with this assumption is slightly different from the exact optimal point obtained from the buckling formulas. Stacking sequence optimizations were performed with the GA using the obtained response surface. Table 2 shows the obtained optimal stacking sequence with the GA. We conducted 100 runs of the GA changing the seeds of the random number because the GA is based on random numbers; we obtained a measure of reliability of the optimality. The result indicated 100% reliability in this case. On the basis of the result, we can conclude that the response surface obtained from the D-optimal set of laminates is effective for stacking sequence optimizations.

7. Concluding remarks

This study presented a new D-optimal set of laminates for creation of a response surface for stacking sequence optimizations. The optimal set of laminates can be applied to any structural analysis tools that require practical stacking sequences for calculations because the new D-optimal set of laminates provides optimal stacking sequences for calculations to create a response surface. The proposed D-optimal set of laminates is not limited to 16-ply laminates. It is applicable to laminates of any number of plies by changing the ply thickness. The effectiveness of the method is shown with stacking sequence optimization to maximize the buckling load of an axially compressed cylindrical shell.

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