Key Engineering Materials Vols. 270-273 (2004) pp. 1859-1865
online at http://www.scientific.net

Delamination Identification of CFRP Structure by Discriminant Analysis using Mahalanobis Distance

Atsushi Iwasaki[1] and Akira Todoroki[2]

[1] University of Tokyo, Department of Mechanical Sciences and engineering, 7-3-1 Hongo, Bunkyo-ku Tokyo 113-8656, JAPAN

[2] Tokyo Institute of Technology, Department of Mechanical Sciences and engineering, 2-12-1,Ohokayama, Meguro-ku, Tokyo, 152-8552, Japan

Keywords: Mahalanobis distance, Discriminant analysis, Composites, Electric potential method, Delamination, Structural health monitoring.

Abstract. The present paper proposes a new diagnostic tool for structural health monitoring that employs Mahalanobis Distance for discriminant analyses. In general, the relation between the damage and measurement values is identified by solving an inverse problem. The inverse problem in damage identification is one of the optimization problems involving the minimization of estimation error, and is usually solved by common optimization tools such as neural network and genetic algorithm. However, trial and error processes are generally required to derive the optimum solution using these methods, and these processes demand much calculation and human cost. The present method does not require the trial and error process in order to identify the damage level.

In the present paper, this method is applied to the identification of a delamination crack in a carbon fiber reinforced plastic(CFRP) beam. A delamination crack is invisible and causes a decrease of compression strength of laminated composites. Therefore, a structural health monitoring system is required for CFRP laminates. The present study adopts an electric potential method for monitoring the structural integrity of laminated composites. The electric potential method does not reduce the composite's strength and can be applied to existing structures at low cost. Test results show that this method is effective for delamination identification.

Introduction

Carbon fiber reinforced plastic (CFRP) has high specific tensile strength and stiffness, and are good for the major structures of airplanes and space instruments. The intensity between the layers of CFRP is weak and delamination cracks between layers can be produced by a comparatively slight impact. Such delamination cracks are invisible and cause a decrease of the compression strength of laminated composites. Therefore, a method for diagnosing delamination cracks in laminated composites is required. The present study adopts an electrical potential method for the diagnosis of delaminations in CFRP laminates. Since carbon fiber in a composite material has conductivity, by using an electric potential method, the damage present in a large area of a structure can be diagnosed by conducting electricity through composite structure. The electric potential method does not cause strength reduction and can be applied to existing structures at a low cost. The present study focuses on the electrical potential method of detecting damage in CFRP in the form of delamination cracks.

Recently, many studies have described an electric potential change method to identify internal damage in CFRP laminates [1-18]. The electric potential change method does not require expensive instruments. Since the method adopts reinforced carbon fiber itself as sensors for damage detection, this method does not cause a reduction in static strength or fatigue strength, and it is applicable to existing structures. The effectiveness of the electric potential method has been demonstrated in our previous reports [19-28]. In these studies, a delamination crack in a composite beam was identified based on the change in electric potential between electrodes attached to the surface of the composite beam. The relation between the delamination crack and the change in electric potential is identified by solving an inverse problem. The inverse problem in damage identification is an optimization problems involving estimation error minimization, and is usually solved through the use of such common optimization tools as neural network and genetic algorithm. But in general, trial and error processes are required to derive optimum solutions when using these methods, and these processes demand much calculation and human cost.

We thus propose a method that identifies damage based on statistical analyses for the purpose of constructing a simple method for damage identification. In this method, we use discriminant analysis based on Mahalanobis distance as a statistical distance indicator. Using discriminant analysis, damage identification that requires only simple calculation can be performed.

In this study, this new method is applied to identify delamination cracks in CFRP beams using the electric potential method. The effectiveness of this method is investigated analytically.

Discriminant analysis based on Mahalanobis distance

Discriminant analysis is a statistical method used to distinguish a group to which an individual belongs. Mahalanobis distance [29] is an effective statistical indicator for discriminant analysis when the variance of each group is different. Discriminant analysis based on Mahalanobis distance demands complicated calculation compared with the case in which linear discriminant function is used. But if a variance-covariance matrix of each group is different, the discriminant analysis using Mahalanobis distance achieves a higher degree of accuracy than does linear discriminant function. Mahalanobis distance is the distance between an individual and the centroid for each group. An individual has one Mahalanobis distance for each group, and it will be classified into the group for which its Mahalanobis distance is minimum. The data groups which are created from data for learning are called standard spaces.

Generally, the Mahalanobis distance from a standard space i is represented as following formula.

$$D_i = \frac{1}{T}\sum_{l=1}^{T}\sum_{k=1}^{T}\left(X_{il} - \overline{X_{il}}\right)S_{lk}^{-1}\left(X_{ik} - \overline{X_{ik}}\right) \tag{1}$$

where D_i is Mahalanobis distance from the standard space i. X_{ij} is the standardization of predictor valuable x_{ij}(i: 1～N, j: 1～T) derived as following formula.

$$X_{ij} = \frac{x_{ij} - \overline{x}_j}{\sigma_{x_j}} \tag{2}$$

where N is the number of standard spaces and T is the freedom of data. $\boldsymbol{S}$ is the standardized variance-covariance matrix derived as following formula.

$$S_{lk} = \frac{1}{T}\sum_{i=1}^{T}\left(X_{il} - \overline{X}_l\right)\left(X_{ik} - \overline{X}_k\right) \tag{3}$$

where $\overline{X_l}$ is the average of X_{il} (i: 1～n), where n is number of data for the learning of the standard space i.

The Mahalanobis distance becomes smaller value when the individual is near the standard space, and takes a larger value when it is far from the standard space. Because of this tendency, the data is diagnosed as belonging to the standard space that takes the minimum Mahalanobis distance.

Test of effectiveness of diagnosis using χ^2-distribution. Since the Mahalanobis distance to an actual group(= actual standard space) follows the χ^2-distribution, the effectiveness of diagnosis is assessed based on the results of the χ^2 test.

The probability density of Mahalanobis distance to an actual group follows the χ^2-distribution, and the probability density function is defined as following formula.

$$p_i = \frac{1}{2^{(T/2)}\Gamma(T/2)} x^{\left(\frac{T}{2}\right)-1} \cdot e^{-\frac{1}{2}D_i} \tag{4}$$

where p_i is probability density. Since the distance follows the χ^2 distribution, the critical value of the χ^2 test indicates the threshold value of the effectiveness of the diagnosis. When the distance to the estimated group exceeds the critical value of the χ^2 test, the effectiveness of the estimation is rejected and re-measurement of estimated data is required.

Identification of delaminations in a CFRP beam using Electric Potential method

Analytical model. As mentioned previously, this new method for damage diagnosis is applied to the identification of delaminations in a CFRP Beam, and the accuracy of the method is experimentally investigated.

FEM analyses are employed for investigations in present study. A detailed description of FEM analysis is provided in our previous studies [22,23]. The configuration of the specimen used in the present study is shown in Figure 1. The specimen is a CFRP Beam with a thickness of 2mm and a stacking sequence of $[0_2/90_2]$s. In order to measure the change in electric potential caused by a delamination crack, seven electrodes are mounted on the one surface of specimen. The lengths of the electrodes are 10mm.

The FEM analyses were performed using the commercially available FEM tool named ANSYS. In the present study, four-node-rectangular elements are adopted for analysis; each element is approximately 0.125 mm by 0.125 mm. Total number of the elements is 28,160. A delamination crack is modeled by means of the release of a nodal point of the element. The electric conductance ratio is obtained from an experimental result regarding a CFRP laminate whose volume fraction is 62% as shown in Table 1.

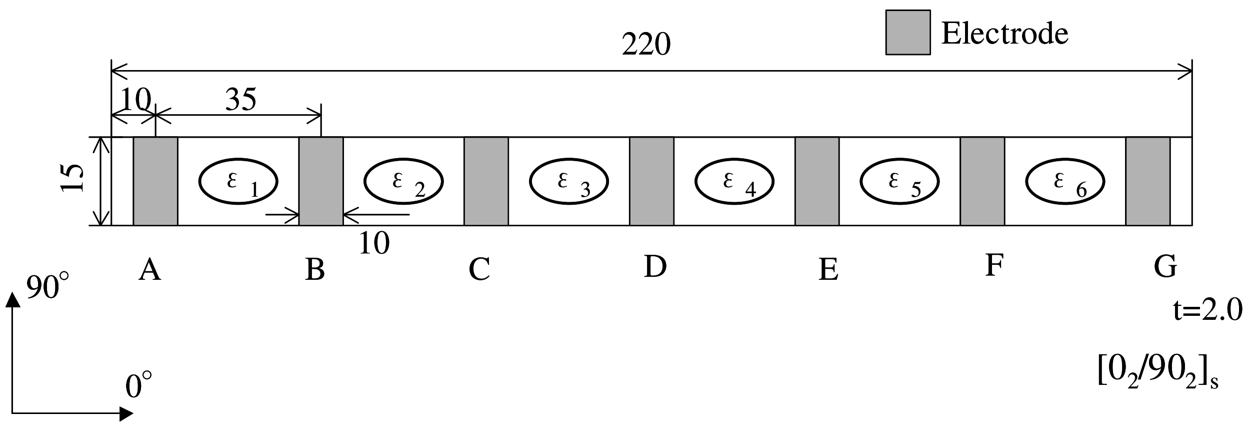

Figure 1. Specimen configuration

Table 1. Electric conductivity of the specimen

$\sigma_{0°}$	$5535[\Omega^{-1}]$
$\sigma_{90°}/\sigma_{0°}$	3.71×10^{-2}
$\sigma_{tckness}/\sigma_{0°}$	3.77×10^{-3}

Procedure for identifying delamination crack

The procedure for identifying a delamination crack is as follows: (1) to (3) show the procedure for creating a standard spaces and (4) to (7) show the procedure for the identification of individual.

(1) The change in electric potential is measured in the various sizes and locations of delamination cracks. These data are referred to as data for learning.
(2) The location and sizes of the delamination crack are defined, and the standard space of each level is created based on the data for learning.
(3) The variance-covariance matrix of the standard space is calculated.
(4) New data is measured for estimation. This data is referred to as individual.
(5) The Mahalanobis distances to the standard spaces of estimating data are calculated.
(6) The group to which the estimating data belongs is determined using Mahalanobis distances.
(7) Effectiveness of estimation is tested using the χ^2 test. When effectiveness is rejected, return to (4) and regather the data regarding the current condition.

In this study, cross inspections of 474 data are conducted. One data is selected as individual, and standard spaces are created from other 473 data. Experimental error of an average of 0 and a distribution of 0.01 are added as measurement error for estimating data. This procedure is performed 474 times.

As shown in Table 2, each data set consists of the location and size of the delamination crack, and six electric potential change ratios between electrodes ε_i (i=1-6). The electric potential change ratios are normalized by the norm of the measured electric voltage change vector η. The norm of the measured electric voltage change vector and normalized electric potential changes vector are as following formula.

$$\eta = \sqrt{\sum_{i=1}^{6} \varepsilon_i^{\ 2}} \tag{5}$$

$$\begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_6 \end{pmatrix} = \begin{pmatrix} \varepsilon_1/\eta \\ \varepsilon_2/\eta \\ \vdots \\ \varepsilon_6/\eta \end{pmatrix} \tag{6}$$

The Mahalanobis distance is derived from η and v_i (i=1-6).

The location and size level of the delamination crack are defined in Table 3. The location of x direction is divided to six levels and size is divided to three levels. The number of data, which are divided into each level, is shown in Table 4.

Table 2. Data for estimation

Test No.	Location [mm]	Size [mm]	Electric potencial change ratio ε_1 (AB)	ε_2 (BC)	ε_3 (CD)	ε_4 (DE)	ε_5 (EF)	ε_6 (FG)
1	31	20	0.00E+00	0.00E+00	3.96E-02	4.70E+00	5.03E+00	1.09E-02
2	-16	30	5.47E-03	1.62E+00	1.95E+00	2.86E+00	0.00E+00	0.00E+00
3	-37	40	2.68E-01	1.19E+01	1.26E+01	1.25E-01	0.00E+00	0.00E+00
⋮	⋮	⋮					⋮	
⋮	⋮	⋮					⋮	
474	1	40	0.00E+00	1.47E-01	1.25E+01	1.21E+01	2.26E-01	0.00E+00

Table 3. Location and size level of delamination

	Level [mm] 1	2	3	4	5	6
x	$0 \leqq x < 35$	$35 \leqq x < 70$	$70 \leqq x < 105$	$105 \leqq x < 140$	$140 \leqq x < 175$	$175 \leqq x < 210$
Size	2	10	20	30	40	

Table 4. Distributions of measured data

	Level					
	1	2	3	4	5	6
x	62	88	87	87	88	62
Size	101	101	96	90	86	---

Results and Discussions

Delamination diagnosis without effectiveness test

Identification of location of delamination crack. The Mahalanobis distance to the standard spaces of each location level has calculated first. Table 5 shows the average Mahalanobis distance to each standard space. The average Mahalanobis distances to the actual levels are evidently smaller than that to other location levels as shown in Table 5. This tendency appears in both data for learning and that for estimating. The Mahalanobis distance to actual level is short at the location of the delamination crack.

Table 6 shows the reliability of the estimated locations of the delamination cracks. The diagnostic accuracy of locating the delamination is 84.9 percent for the data for learning and is 79.5 percent for the estimating data. These results show that the new diagnostic method achieves a high level of performance in locating delamination cracks.

Table 5. Average of Mahalanobis Distance (Location)

	Average of Mahalanobis Distance(MD)	
	MD from actual location level	MD from other location level
Data for learning	6.911	2.282E+08
Estimating data	8.370	2.249E+08

Table 6. Reliability of location estimation

		Reliability of Estimation
Data for learning	*x*	84.9%
Estimating data	*x*	79.5%

Identification of the size of the delamination crack. Table 7 shows the average Mahalanobis distance. As shown in the table, the difference between actual and other location levels is not clear compared with that of the location identification This means that the identification of size is more difficult than the identification of location. Table 8 shows the reliability of the estimation of the size of delamination cracks using this method. The diagnostic accuracy of delamination size is 74.4 percent for the data for learning and is 68.1 percent for estimating data. As shown in Table 7, the average value in relation to the actual level is sufficiently smaller than the average value in relation to other levels, and it is considered that diagnostic accuracy is improvable by increasing the number of the data for the standard spaces.

Table 7. Average Mahalanobis Distance (Size)

	Average of Mahalanobis Distance(MD)	
	MD from actual location level	MD from other location level
Data for learning	6.926	3.852E+04
Estimating data	7.708	3.822E+04

Table 8. Reliability of size estimation

		Reliability of Estimation
Data for learning	Size	74.4%
Estimating data	Size	68.1%

Delamination identification with effectiveness test of estimation. When the effectiveness is rejected, error of an average of 0 and a distribution 0.01 was added as a new experimental error to the estimating data. As shown in Table 9, for the estimating data, the diagnostic accuracy regarding location is 86.9% and the accuracy regarding size is 79.6%. These performance results show that the method shows high generalization capability to identify the size and location of delamination cracks and the accuracy of delamination identification of the composite beam using new diagnostic method is improved by using effectiveness test of estimation.

Table 9. Reliability of estimation

	Reliability of Estimation	
	Location	Size
Data for learning	90.0%	84.9%
Estimating data	86.9%	79.6%

Conclusions

The present paper describes a new method using simple calculation for the identification of delamination cracks in CFRP beams through the application of discriminant analysis based on Mahalanobis distance. The main conclusions are as follows:

(1) Discriminant analysis using Mahalanobis distance shows high accuracy in identifying delamination in CFRP beams using the electric potential method.
(2) Damage diagnosis using the effectiveness test regarding estimation shows a generalization capability to identify the size and location of delamination cracks.

References

[1] Moriya K., and Endo T., Aeronautical and Space Science Japan. 1988;36(410);139-146 (in Japanese).

[2] Schulte K. and Baron Ch., Composite Science and Technology, 1989;36; 63-76.

[3] Muto N., Yanagida H., Miyayama M., Nakatsuji T., Sugita M. and Ohtsuka Y., J. of the Japan Society for Composite Materials. 1992;18(4);144-150 (in Japanese).

[4] Fischer Chr. and Arendts F.J., Composite Science and Technology.1993; 46;319-323.

[5] Chen P.W., and Chung D.D.L., Smart Mater. Struct,.1993;2;22-30.

[6] Kaddoour A.S., Al-Salehi F.A. and Al-Hassani S.T.S., Composite Science and Technology.1994;51;377-385.

[7] Wolfiger C., Drechsier K., ESTEC, Noordwijk (ESA-WPP-070).1994;677-782.

[8] Chen P.W., and Chung D.D.L., J.Am.Ceram.Soc..1995; 78(3);816-818.

[9] Wang X. and Chung D.D.L., Polymer Composites.1997;18(6); 692-700.

[10] Irving P.E. and Thiagarajan C., Smart Materials and Structures. 1998;7;456-466.

[11] Abry J.C., Bochard S., Chateauminois A., Salvia M. and Giraud G., Composite Science and Technology. 1999;59;925-935.

Title of Publication (to be inserted by the publisher)

[12] Seo D.C. and Lee J.J., Composite Structures. 1999;47;525-530.

[13] Abry J.C., Choi Y.K., Chateuminois A. Dalloz B. and Giraud G., Composite Science and Technology. 2001;61;855-864.

[14] Weber I., Schwartz P., Composite Science and Technology; 2001;61;849-853.

[15] Muto N., Arai Y., Shin S.G., Matsubara H. Yanagaida H., Sugita M., and Nakatsuji T., Composite Science and Technology. 2001;61;875-883.

[16] Kupke M., Schulte K. and Schuler R., Composite Science and Technology. 2001;61;837-847.

[17] Schueler R., Joshi S.P. and Schulte K., Composite Science and Technology. 2001;61;921-930.

[18] Kubo S., Kuchinishi M. Sakagami T. and Ioka S., Applied Electromagnetics and Mechanics, ed. T. Takagi and M. Uesaka, Proc. of the 10th Int. Symp. on Applied Electromagnetics and Mechanics, Japan Soc. Applied Electromagnetics and Mechanics. 2001;641-642.

[19] Todoroki A., Matsuura K. and Kobayashi H., JSME International J., Series A. 1995;38(4) ;524-530.

[20] Todoroki A., Kobayashi H. and Matsuura, K., US-Japan Workshop on Smart Materials and Structures, Edited by K.Inoue, S.I.Y.Shen and M.Taya, University of Washington TMS. 1997;47-54 .

[21] Todoroki A., Proceedings of the 5th Japan International SAMPE Symposium;1997;899-904.

[22] Todoroki A., Suzuki H., Kobayashi H., Nakamura H., and Shimamura Y., Transactions of the Japan Society of Mechanical Engineers Series A. 1998; 64(622);1654-1659. (in Japanese).

[23] Todoroki A. and Suzuki H., Applied Mechanics and Engineering.2000; 5(1);283-294.

[24] Todoroki A., Tanaka Y., and Shimamura Y., Structural Health Monitoring 2000, edited by F.K.Chang, Technomic. 1999;308-316.

[25] Todoroki A., Tanaka Y., and Shimamura Y., Materials Science Research International, Special Technical Publication-2, JSMS. 2001;139-145.

[26] Todoroki, A., Tanaka, M., and Shimamura, Y., Composite Science and Technology ,62,5, (2002), p.619-628.

[27] Todoroki A., Composite Science and Technology, 2001;61(13);1871-1880.

[28] Todoroki A., Tanaka M, and Shimamura Y., Composite Science and Technology, (in submitting).

[29] D.Maesschalck, D.Jouan-Rimbaud and D.L. Massart, Chemometrics and Intelligent Laboratory Systems, Vol.50, (2000), 1–18.