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Improved Fractal Branch and Bound Method for Stacking-Sequence Optimizations of Laminates

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In the present study, a mechanism of fractal pattern of feasible laminates in lamination parameter coordinates is discussed in detail, and a new improved stacking-sequence optimization method is proposed for complicated structures that include both in-plane and out-of-plane lamination parameters. The new method employs a branch-and-bound method for the optimizations of stacking sequences. For the estimation of the fractal branch of stacking sequences, the new method requires approximation of the objective function of the optimizations with a quadratic polynomial using both in-plane and out-of-plane lamination parameters. The new method is applied to a stacking-sequence optimization problem of a maximization of buckling load of a hat-type stiffener structure. The method gives successfully optimal stacking sequences in an excellently short CPU time.

I. Introduction

IT is well known that stacking-sequence optimizations are indispensable for laminated composite structures. Laminated composite structures are usually made from unidirectional plies of given thickness and with fiber orientations limited to a small number of fiber angles, such as 0-, ±45-, and 90-deg plies. The problem of designing such laminates for various strength and stiffness requirements is an integer-programming problem of selecting the required number of plies of each orientation and deciding on the optimal stacking sequence. Although the branch-and-bound method has been occasionally adopted for solution (e.g., Ref. 1), genetic algorithms have been very popular for solving such problems (e.g., Refs. 2–22).

For the genetic algorithms for optimizations of stacking sequences, however, constraints of stacking sequences such as the limit of number of contiguous plies of the same fiber orientation are difficult to implement, and the genetic algorithms require large computational resources for evaluations of individual laminated structures. Le Riche and Haftka[3] have introduced a couple of plies to implement the stacking-sequence constraint of a balance rule of angle plies for the genetic algorithm. Harrison et al.[8] introduced trinal numbers for genetic coding of stacking sequences to implement the balance rule. Todoroki and Haftka[16,17,19] introduced repair strategies of genetic coding to implement the balance rule and the four-contiguous ply rule. Liu et al. also apply the repair strategy to implement the constraint of ratio of fiber orientations.[21]

To reduce the computational cost of the genetic algorithm for the stacking-sequence optimizations, response surfaces are employed for reduction of evaluation cost.[8] Todoroki and Haftka[19] used a two-stage response surface method to obtain an approximated objective function. Because the two-stage response surface method adopts lamination parameters introduced by Miki[23] as variables of the response surfaces, the number of variables is independent of the number of plies, and quadratic polynomials are enough to approximate entire objective function. These methods reduce the computational cost of the genetic algorithms for stacking-sequence optimizations. The genetic algorithm, however, still requires a large number of adjustments to obtain high performance, and the method cannot always obtain the real optimal stacking sequence because the method is one of the probabilistic approaches.

Authors[24] have revealed that plotting feasible laminates in out-of-plane lamination parameters creates a self-similar fractal pattern. An example of the fractal pattern of an entire set of symmetric laminates of 12 plies comprising only 0-, ±45-, and 90-deg plies is shown in Fig. 1. In this figure the abscissa is the first out-of-plane lamination parameter W_1^*, and the ordinate is the second out-of-plane lamination parameter W_2^*. All dots represent coordinate values of feasible laminates. On the basis of the fractal branches of the feasible laminates, we have proposed a novel optimizing method named fractal branch-and-bound method. The paper provides a mathematical background why the set of laminates creates fractal pattern when they are plotted in the coordinates of the out-of-plane lamination parameters. With the approximation of the objective function using quadratic polynomials in out-of-plane lamination parameters, a branch-and-bound method to obtain a set of neighbor laminates to the optimal point is performed to obtain an optimal stacking sequence.

Because the proposed method is limited to problems that require only the out-of-plane lamination parameters, an improvement of the method has been required for more complicated laminated structures such as stiffened panels including both in-plane and out-of-plane lamination parameters. In the present paper, therefore, an improved version of the fractal branch-and-bound method is proposed, and the method is applied to a problem maximizing buckling load of a hat-type stiffener. It is possible to apply the method on any problems including both in-plane and out-of-plane lamination parameters. Similar to the previous paper, the method employs the response surface approximation of an objective function using quadratic polynomials in both of lamination parameters.

II. Stacking-Sequence Optimization

A. Lamination Parameters

In-plane stiffness terms of the symmetric laminates are represented with in-plane lamination parameters $\boldsymbol{V}_i^*$ $(i = 1, 2, 3, 4)$ as refereed in some texts (e.g., Ref. 25). The in-plane lamination parameters are given as follows:

$$\boldsymbol{V} = \begin{bmatrix} V_1^* \\ V_2^* \\ V_3^* \\ V_4^* \end{bmatrix} = \frac{2}{h}\int_0^{h/2} \begin{bmatrix} \cos 2\theta \\ \cos 4\theta \\ \sin 2\theta \\ \sin 4\theta \end{bmatrix} \mathrm{d}z \qquad (1)$$

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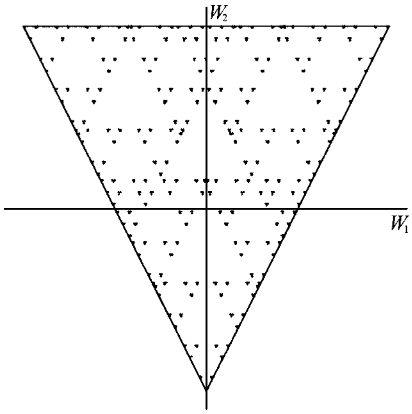


Fig. 1 Fractal pattern feasible laminates in out-of-plane lamination parameter drawn by plotting feasible laminates of 10 plies (*N* = 5).

where h is the thickness of the laminate, z is a coordinate of thickness direction, the origin locates in the middle of the plate, and $\theta(z)$ is the fiber angle of the location of z.

The out-of-plane stiffness terms of the laminates are represented with out-of-plane lamination parameters $\boldsymbol{W}_i^*$, and the out-of-plane lamination parameters are defined as follows:

$$\boldsymbol{W} = \begin{bmatrix} W_1^* \\ W_2^* \\ W_3^* \\ W_4^* \end{bmatrix} = \frac{24}{h^3}\int_0^{h/2} z^2 \begin{bmatrix} \cos 2\theta(z) \\ \cos 4\theta(z) \\ \sin 2\theta(z) \\ \sin 4\theta(z) \end{bmatrix} \mathrm{d}z \tag{2}$$

Let us consider the case of the thickness of each ply is t, the integral formulas are rewritten as summations by using the half-number of plies N, and the fiber angle of the kth ply θ_k from the outer ply:

$$h = 2tN \tag{3}$$

$$\boldsymbol{V} = \frac{1}{tN}\sum_{k=1}^{N}\int_{(N-k)t}^{(N-k+1)t} \begin{bmatrix} \cos 2\theta \\ \cos 4\theta \\ \sin 2\theta \\ \sin 4\theta \end{bmatrix} \mathrm{d}z$$

$$= \frac{1}{tN}\sum_{k=1}^{N}[(N-k+1)-(N-k)] \begin{bmatrix} \cos 2\theta \\ \cos 4\theta \\ \sin 2\theta \\ \sin 4\theta \end{bmatrix} \tag{4}$$

$$\boldsymbol{W} = \frac{3}{t^3N^3}\sum_{k=1}^{N}\int_{(N-k)t}^{(N-k+1)t} \begin{bmatrix} \cos 2\theta \\ \cos 4\theta \\ \sin 2\theta \\ \sin 4\theta \end{bmatrix} z^2\,\mathrm{d}z$$

$$= \frac{3}{t^3N^3}\sum_{k=1}^{N}[(N-k+1)^3-(N-k)^3] \begin{bmatrix} \cos 2\theta \\ \cos 4\theta \\ \sin 2\theta \\ \sin 4\theta \end{bmatrix} \tag{5}$$

Equations (4) and (5) are rewritten by replacements of the coefficients as follows:

$$a_k^V = \frac{N-k}{N}, \qquad a_k^W = \left(\frac{N-k}{N}\right)^3 \tag{6}$$

$$\boldsymbol{V} = \sum_{k=1}^{N}\left(a_{k-1}^V - a_k^V\right) \begin{bmatrix} \cos 2\theta(z) \\ \cos 4\theta(z) \\ \sin 2\theta(z) \\ \sin 4\theta(z) \end{bmatrix} \tag{7}$$

$$\boldsymbol{W} = \sum_{k=1}^{N}\left(a_{k-1}^W - a_k^W\right) \begin{bmatrix} \cos 2\theta(z) \\ \cos 4\theta(z) \\ \sin 2\theta(z) \\ \sin 4\theta(z) \end{bmatrix} \tag{8}$$

For most of practical laminated composite structures, fiber angle is limited to small set such as 0, 45, −45, and 90 deg because of lack of experimental data. Let us consider the case that the fiber angle is limited to the set. All values of the trigonometric functions in Eqs. (7) and (8) are listed as follows:

$$\begin{bmatrix} \cos 2\theta_k \\ \cos 4\theta_k \\ \sin 2\theta_k \\ \sin 4\theta_k \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \text{for 0-deg ply}, \quad = \begin{bmatrix} 0 \\ -1 \\ 1 \\ 0 \end{bmatrix} \text{for 45-deg ply}$$

$$= \begin{bmatrix} -1 \\ -1 \\ 0 \\ 0 \end{bmatrix} \text{for 90-deg ply}, \quad = \begin{bmatrix} 0 \\ -1 \\ -1 \\ 0 \end{bmatrix} \text{for −45-deg ply} \tag{9}$$

Because the value of $\sin 4\theta_k$ is always zero, the values of V_4^* and W_4^* are always zero. This makes the reduced lamination parameter vectors of three dimensions as follows:

$$\boldsymbol{V} = \sum_{k=1}^{N}\left(a_{k-1}^V - a_k^V\right) \begin{bmatrix} \cos 2\theta(z) \\ \cos 4\theta(z) \\ \sin 2\theta(z) \end{bmatrix} \tag{10}$$

$$\boldsymbol{W} = \sum_{k=1}^{N}\left(a_{k-1}^W - a_k^W\right) \begin{bmatrix} \cos 2\theta(z) \\ \cos 4\theta(z) \\ \sin 2\theta(z) \end{bmatrix} \tag{11}$$

Because the values of all trigonometric functions are written in Eqs. (9), any lamination parameter sets can be written by linear summations of four vectors of each fiber angle as follows:

$$\boldsymbol{V} = s_0^V \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + s_{45}^V \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} + s_{90}^V \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + s_{-45}^V \begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix} \tag{12}$$

$$\boldsymbol{W} = s_0^W \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + s_{45}^W \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} + s_{90}^W \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + s_{-45}^W \begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix} \tag{13}$$

where s is the coefficients defined as follows:

$$s_j^V = \sum_{k=1}^{N}\delta\left(a_{k-1}^V - a_k^V\right)$$

$$\delta = 1 \quad \text{when} \quad \theta_k = \theta_j \ (j = 0, 45, -45, \text{and } 90 \text{ deg})$$

$$\delta = 0 \quad \text{when} \quad \theta_k \neq \theta_j \ (j = 0, 45, -45, \text{and } 90 \text{ deg}) \tag{14}$$

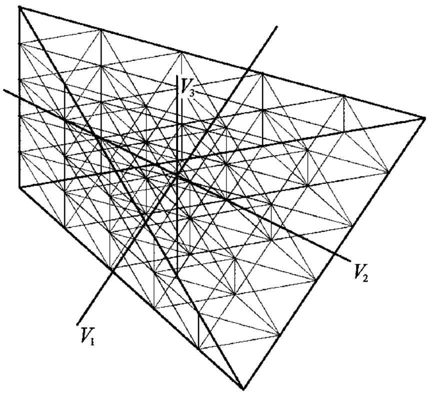
$$s_j^W = \sum_{k=1}^{N}\delta\left(a_{k-1}^W - a_k^W\right) \tag{15}$$

The coefficient s satisfies the following equations from Eq (6):

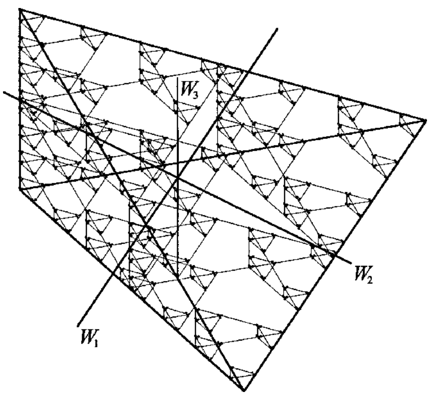
$$0 \le s_j^V \le 1, \qquad 0 \le s_j^W \le 1 \tag{16}$$

$$s_0^V + s_{45}^V + s_{-45}^V + s_{90}^V = 1, \qquad s_0^W + s_{45}^W + s_{-45}^W + s_{90}^W = 1 \tag{17}$$

Equation (16) represents that any vector of lamination parameter $\boldsymbol{V}$ and $\boldsymbol{W}$ can be plotted inside of a tetrahedron defined by four vectors: (1,1,0), (0,−1,1),(−1,1,0), and (0,−1,−1).



a) In-plane lamination parameter coordinates



b) Out-of-plane lamination parameter coordinates

Fig. 2 Fractal tetrahedron pattern of feasible laminates drawn by plotting all feasible laminates of 10 plies ($N = 5$) in lamination parameters.

B. Creation of Fractal Pattern of Feasible Laminates

When all of feasible laminates are plotted, the plots create fractal pattern as shown in Fig. 2. Figure 2 is the case of the laminate of $N = 5$ (total number of plies is 10). As you can see, the plots of $\boldsymbol{W}$ show a fractal pattern the same as Fig. 1. Let us explain why the plots creates a fractal pattern in the case of $N = 5$.

In the case of $N = 5$ (10 plies), any stacking sequences can be written as follows:

$$[\theta_1/\theta_2/\theta_3/\theta_4/\theta_5]s$$

When the outermost ply is decided to 0-deg ply, all branches of the feasible laminates that start from the outermost 0-deg ply can be written as follows:

$$\boldsymbol{V} = \left(a_0^V - a_1^V\right)\begin{bmatrix}1\\1\\0\end{bmatrix} + \sum_{k=2}^{5}\left(a_{k-1}^V - a_k^V\right)\begin{bmatrix}\cos 2\theta(z)\\ \cos 4\theta(z)\\ \sin 2\theta(z)\end{bmatrix} \tag{18}$$

$$\boldsymbol{W} = \left(a_0^W - a_1^W\right)\begin{bmatrix}1\\1\\0\end{bmatrix} + \sum_{k=2}^{5}\left(a_{k-1}^W - a_k^W\right)\begin{bmatrix}\cos 2\theta(z)\\ \cos 4\theta(z)\\ \sin 2\theta(z)\end{bmatrix} \tag{19}$$

We can redefine the coefficient s of Eqs. (14) and (15) as follows:

$$s_j^V = \sum_{k=2}^{5}\delta\left(a_{k-1}^V - a_k^V\right) \tag{20}$$

$$s_j^W = \sum_{k=2}^{5}\delta\left(a_{k-1}^W - a_k^W\right) \tag{21}$$

Equations (20) and (21) bring the following equations:

$$0 \le s_j^V \le 1, \qquad 0 \le s_j^W \le 1 \tag{22}$$

$$s_0^V + s_{45}^V + s_{-45}^V + s_{90}^V = a_1^V, \qquad s_0^W + s_{45}^W + s_{-45}^W + s_{90}^W = a_1^W \tag{23}$$

Let us rewrite the coefficient s as follows:

$$p_j^V = \left(1/a_1^V\right)s_j^V, \qquad p_j^W = \left(1/a_1^W\right)s_j^W \tag{24}$$

The coefficient p satisfies the following equations:

$$0 \le p_j^V, \qquad 0 \le p_j^W \tag{25}$$

$$p_0^V + p_{45}^V + p_{-45}^V + p_{90}^V = 1, \qquad p_0^W + p_{45}^W + p_{-45}^W + p_{90}^W = 1 \tag{26}$$

Using the coefficients p just defined, Eqs. (18) and (19) can be rewritten as follows:

$$\boldsymbol{V} = \boldsymbol{V}_0 + a_1^V\left\{p_0^V\begin{bmatrix}1\\1\\0\end{bmatrix} + p_{45}^V\begin{bmatrix}0\\-1\\1\end{bmatrix} + p_{-45}^V\begin{bmatrix}0\\-1\\-1\end{bmatrix} + p_{90}^V\begin{bmatrix}-1\\1\\0\end{bmatrix}\right\} \tag{27}$$

$$\boldsymbol{W} = \boldsymbol{W}_0 + a_1^W\left\{p_0^W\begin{bmatrix}1\\1\\0\end{bmatrix} + p_{45}^W\begin{bmatrix}0\\-1\\1\end{bmatrix} + p_{-45}^W\begin{bmatrix}0\\-1\\-1\end{bmatrix} + p_{90}^W\begin{bmatrix}-1\\1\\0\end{bmatrix}\right\} \tag{28}$$

where

$$\boldsymbol{V}_0 = \left(a_0^V - a_1^V\right)\begin{bmatrix}1\\1\\0\end{bmatrix}, \qquad \boldsymbol{W}_0 = \left(a_0^W - a_1^W\right)\begin{bmatrix}1\\1\\0\end{bmatrix} \tag{29}$$

Equations (27) and (28) construct a self-similar shrunk tetrahedron by the factor of a_1^V $(= 4/5 = 0.8)$ or a_1^W $(= 64/125 = 0.512)$. The apexes of the tetrahedron are as follows:

$$\boldsymbol{V}_0 + a_1^V\begin{bmatrix}1\\1\\0\end{bmatrix}, \qquad \boldsymbol{V}_0 + a_1^V\begin{bmatrix}0\\-1\\1\end{bmatrix}$$

$$\boldsymbol{V}_0 + a_1^V\begin{bmatrix}0\\-1\\-1\end{bmatrix}, \qquad \boldsymbol{V}_0 + a_1^V\begin{bmatrix}-1\\1\\0\end{bmatrix} \tag{30}$$

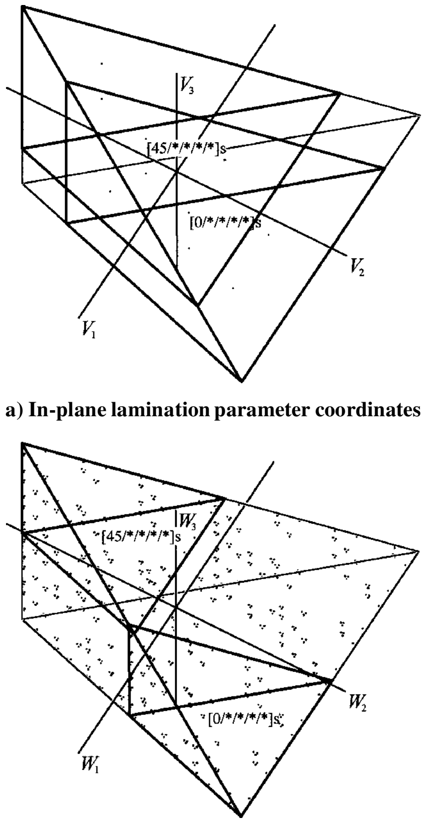
$$\boldsymbol{W}_0 + a_1^W\begin{bmatrix}1\\1\\0\end{bmatrix}, \qquad \boldsymbol{W}_0 + a_1^W\begin{bmatrix}0\\-1\\1\end{bmatrix}$$

$$\boldsymbol{W}_0 + a_1^W\begin{bmatrix}0\\-1\\-1\end{bmatrix}, \qquad \boldsymbol{W}_0 + a_1^W\begin{bmatrix}-1\\1\\0\end{bmatrix} \tag{31}$$

Similarly, we can obtain the tetrahedron that is a collection of laminates that starts from the 45-deg ply. These shrunk tetrahedrons are shown in Fig. 3. In Fig. 3 two tetrahedrons are drawn in each figure. One is the tetrahedron that represents the entire region and includes all feasible laminates whose outermost ply is 0-deg ply. Another is the tetrahedron that the represents entire region and includes all feasible laminates whose outermost ply is 45-deg ply.

Let us consider the case that the second outer ply is decided to 90-deg ply. All of the branches of the stacking sequence are represented as $[0/90/\theta_3/\theta_4/\theta_5]$s. In this case Eqs. (18) and (19) are rewritten as follows:

$$\boldsymbol{V} = \left(a_0^V - a_1^V\right)\begin{bmatrix}1\\1\\0\end{bmatrix} + \left(a_1^V - a_2^V\right)\begin{bmatrix}-1\\1\\0\end{bmatrix} + \sum_{k=3}^{5}\left(a_{k-1}^V - a_k^V\right)\begin{bmatrix}\cos 2\theta(z)\\ \cos 4\theta(z)\\ \sin 2\theta(z)\end{bmatrix} \tag{32}$$



a) In-plane lamination parameter coordinates

b) Out-of-plane lamination parameter coordinates

Fig. 3 Fractal shrinking mechanism of tetrahedron of feasible laminates ($N = 5$).

$$W = \left(a_0^W - a_1^W\right)\begin{bmatrix}1\\1\\0\end{bmatrix} + \left(a_1^W - a_2^W\right)\begin{bmatrix}-1\\1\\0\end{bmatrix} + \sum_{k=3}^{5}\left(a_{k-1}^W - a_k^W\right)\begin{bmatrix}\cos 2\theta(z)\\ \cos 4\theta(z)\\ \sin 2\theta(z)\end{bmatrix} \tag{33}$$

As the same as the preceding, we can rewrite the equations as follows:

$$V = V_0 + a_1^V\left\{p_0^V\begin{bmatrix}1\\1\\0\end{bmatrix} + p_{45}^V\begin{bmatrix}0\\-1\\1\end{bmatrix} + p_{-45}^V\begin{bmatrix}0\\-1\\-1\end{bmatrix} + p_{90}^V\begin{bmatrix}-1\\1\\0\end{bmatrix}\right\} \tag{34}$$

$$W = W_0 + a_1^W\left\{p_0^W\begin{bmatrix}1\\1\\0\end{bmatrix} + p_{45}^W\begin{bmatrix}0\\-1\\1\end{bmatrix} + p_{-45}^W\begin{bmatrix}0\\-1\\-1\end{bmatrix} + p_{90}^W\begin{bmatrix}-1\\1\\0\end{bmatrix}\right\} \tag{35}$$

where

$$V_0 = \left(a_0^V - a_1^V\right)\begin{bmatrix}1\\1\\0\end{bmatrix} + \left(a_1^V - a_2^V\right)\begin{bmatrix}-1\\1\\0\end{bmatrix}$$

$$W_0 = \left(a_0^W - a_1^W\right)\begin{bmatrix}1\\1\\0\end{bmatrix} + \left(a_1^W - a_2^W\right)\begin{bmatrix}-1\\1\\0\end{bmatrix} \tag{36}$$

These equations create new self-similar tetrahedrons of which the center points are moved to V_0 or W_0. These are self-similar shrunk tetrahedrons by the factor of a_1^V $(= 3/5 = 0.6)$ or a_1^W $(= 27/125 = 0.216)$. Similarly, the self-similar smaller tetrahedrons are created inside of the tetrahedron after the outer-ply angle is decided. The process makes a fractal pattern of all feasible laminates on the bending lamination parameter vector W. In this case $(N = 5)$ the fractal dimension is 2.23, and the fractal dimension depends on the half-number of plies N.

C. Response Surface Approximation

Response surface methodology is applied to obtain an approximation to a response function in terms of predictor variables.[26] For a quadratic polynomial the response surface is written as follows:

$$y = \beta_0 + \sum_{i=1}^{M}\beta_i x_i + \sum_{i=1, j>i}^{M}\beta_{ij}x_i x_j \tag{37}$$

where β is the unknown coefficients, x is predictor variables, and M is the number of predictor variables. The unknown coefficients are estimated with a least-square-error method of a linear multiple regression.

To judge the performance of the approximation of the response surface, the adjusted coefficient of multiple determination R^2_{adj} (R-square-adjusted) is used:

$$R^2_{\text{adj}} = 1 - \frac{SS_E/(n-k-1)}{S_{yy}/(n-1)} \tag{38}$$

where S_{yy} is the total sum of squares, SS_E is square sum of errors, n is the number of experiments, and k is the total number of unkown coefficients in Eq. (37). Each coefficient of the response surface can be tested by using t-statistic.

In the present study the variables are lamination parameters V_i^* and W_i^* $(i = 1, 2, 3)$, and the response y is the objective function that we want to optimize. Our previous study shows that the entire design domain with lamination parameters can be approximated using a quadratic polynomial.[19] Here also, we adopt quadratic polynomials for the function of response surface of any problems concerning about composite stiffness.

Using a matrix form, the response surface can be expressed as follows:

$$f(V, W) = c + [V^T \quad W^T]\begin{bmatrix}b_V\\ b_W\end{bmatrix} + \frac{1}{2}[V^T \quad W^T]\begin{bmatrix}A_{VV} & A_{VW}\\ A_{VW}^T & A_{WW}\end{bmatrix}\begin{bmatrix}V\\ W\end{bmatrix} \tag{39}$$

where c is the constant b and A are the coefficients obtained by the least-squares method as described above.

To obtain the response surface, a set of data points where y is calculated or measured is required. It is desirable to select a good set of data points that maximizes the accuracy of the approximation for a given number of data points. The process of selecting the data points is known as design of experiments (DOE). In the present study D-optimal DOE is employed. Because the V and W are not independent variables and the stacking sequence comprises discrete fiber angles, we cannot perform the DOE by assuming that the V and W are perfectly independent continuous variables. For example, when we conduct DOE with assumption that the V and W are independent continuous variables, there are a lot of infeasible sets of V and W for the practical laminates. The response of the infeasible sets of laminates cannot be obtained because we cannot compute those values. We adopt, therefore, a new DOE using feasible set of the lamination parameters.

For the new DOE all feasible stacking sequences are collected, and these feasible laminates are all used as candidate laminates. The best set of stacking sequences is selected from these candidate laminates using the D-optimal DOE. The new DOE method is called D-optimal laminates here. For the D-optimal DOE we have to decide the number of experiments. The number is empirically decided to be approximately twice of the number of unknown coefficients of the response surfaces. In the present study we design only symmetric balanced laminates comprising fiber-angle set of 0-, ±45-, and

Table 1 D-optimal laminates and calculated buckling load ratio with FEM

Number	Stacking sequence	Buckling ratio
1	[0/0/0/0/0/0/0/0]s	1.057
2	[0/0/0/0/0/0/−45/45]s	1.223
3	[0/0/0/0/0/0/45/−45]s	1.226
4	[0/0/0/0/0/90/45/−45]s	1.735
5	[0/0/0/0/45/−45/45/−45]s	1.666
6	[0/0/0/45/−45/45/−45/90]s	1.994
7	[0/0/0/90/45/−45/0/90]s	2.573
8	[0/0/0/90/45/0/−45/90]s	2.590
9	[0/0/45/−45/45/0/−45/90]s	2.510
10	[0/0/90/0/0/90/0/0]s	2.621
11	[0/0/90/0/90/0/90/90]s	3.216
12	[0/45/−45/90/90/0/90/0]s	2.221
13	[0/45/−45/0/90/90/90/0]s	2.370
14	[0/45/−45/0/90/45/90/−45]s	2.646
15	[0/45/−45/45/0/−45/0/0]s	2.924
16	[0/45/90/90/−45/0/90/90]s	3.171
17	[0/45/90/90/−45/90/0/0]s	3.203
18	[0/45/90/90/−45/90/90/90]s	3.368
19	[0/90/0/45/0/−45/90/90]s	3.360
20	[0/90/0/45/−45/45/−45/0]s	3.663
21	[0/90/0/45/−45/90/90/0]s	3.826
22	[0/90/0/45/−45/45/90/−45]s	3.908
23	[0/90/0/45/−45/90/90/0]s	3.814
24	[0/90/90/45/0/−45/90/0]s	3.368
25	[0/90/90/45/−45/0/0/0]s	2.072
26	[45/0/−45/45/90/0/0/−45]s	3.360
27	[45/−45/90/0/90/0/90/90]s	2.507
28	[45/−45/90/0/90/90/90/0]s	2.864
29	[45/0/90/90/90/−45/90/90]s	3.159
30	[45/0/90/90/90/90/0/−45]s	3.383
31	[45/−45/0/0/90/0/90/0]s	2.161
32	[45/−45/0/45/−45/0/90/0]s	2.038
33	[45/−45/0/45/−45/90/90/90]s	2.775
34	[45/−45/0/45/90/−45/90/0]s	1.849
35	[45/−45/0/45/90/−45/45/−45]s	2.290
36	[45/−45/0/45/90/−45/90/90]s	1.878

90-deg plies. This limitation shows us that the number of variables of the response surfaces of the approximation of the objective function are only four: V_1^*, V_2^*, W_1^*, and W_2^* when we assume that the balance of the angle plies is performed to reduce W_3^*. Because the number of unknown coefficients of the response surface of quadratic polynomials of four variables is 15, 36 laminates are selected here. In the case that the number of stacks is 16 ($N = 8$), the selected 36 D-optimal laminates are shown in Table 1.

D. Branch-and-Bound Method

Let us consider the case that the response surface is given as Eq. (39). Our objective is to find a stacking sequence that maximizes the response here. In the present study we employ a branch-and-bound method. For a branch-and-bound method we need an efficient evaluation function to prune inefficient branches during searching. Search of the best stacking sequence starts from the provisional one: an apex of the tetrahedron such as unidirectional laminate or selected from the candidate laminates generated by random numbers. For example, let us consider the case of $N = 4$ and start from the provisional optimal stacking sequence of [45/−45/45/−45]s.

An entire set of the feasible stacking sequences can be expressed as a large tree as shown in Fig. 4. First, we start the searching of the branch of [0/∗/∗/∗] s: any stacking sequence of the outermost ply is 0-deg ply (Fig. 5). The shrunk tetrahedron of the branch of the set of laminates of [0/∗/∗/∗] s is easily calculated as shown in the preceding section. Let us assume that we can estimate the maximum value of the response in the shrunk tetrahedron. If the estimated maximum value is lower than the provisional one, it means the tetrahedron does not include a better stacking sequence than the provisional one. If the estimated maximum value is larger than the provisional one, it means the tetrahedron might include better stacking sequences. The evaluation of the maximum value of the stacking sequence inside and on the surface of the tetrahedron need

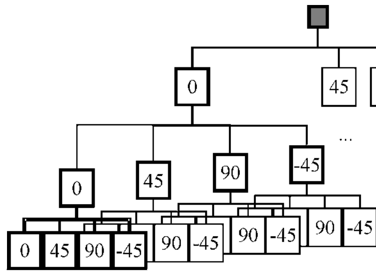


Fig. 4 Fractal branch structure of a stacking-sequence tree.

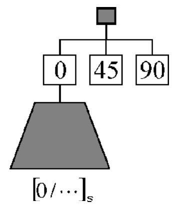


Fig. 5 Branch of stacking sequence whose outermost ply is a 0-deg ply.

not to be exact but must be estimation on the outside. If the evaluated maximum value of the tetrahedron of [0/∗/∗/∗] s is lower than the provisional one, this means that the branch of [0/∗/∗/∗] s, which means any stacking sequence of the outermost 0-deg ply, does not include better stacking sequences, and the branch can be pruned. Similar searching is performed until all branches are investigated.

Let us consider a general case of $[\theta_1/\theta_2/\theta_3/\ldots/\theta_d/*/*/\ldots/*]$s. This means that the outer d plies have been already decided. This case creates a tetrahedron expressed as follows:

$$\boldsymbol{V} = \boldsymbol{V}_0 + a_d^V \boldsymbol{V}', \qquad \boldsymbol{W} = \boldsymbol{W}_0 + a_d^W \boldsymbol{W}' \tag{40}$$

where

$$\boldsymbol{V}_0 = \sum_{k=1}^{d} \left(a_{k-1}^V - a_k^V\right) \begin{bmatrix} \cos 2\theta_k \\ \cos 4\theta_k \\ \sin 2\theta_k \end{bmatrix}$$

$$\boldsymbol{W}_0 = \sum_{k=1}^{d} \left(a_{k-1}^W - a_k^W\right) \begin{bmatrix} \cos 2\theta_k \\ \cos 4\theta_k \\ \sin 2\theta_k \end{bmatrix} \tag{41}$$

$$\boldsymbol{V}' = p_0^V \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + p_{45}^V \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} + p_{-45}^V \begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix} + p_{90}^V \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

$$\boldsymbol{W}' = p_0^W \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + p_{45}^W \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} + p_{-45}^W \begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix} + p_{90}^W \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

$$p_0^V + p_{45}^V + p_{-45}^V + p_{90}^V = 1, \qquad p_0^W + p_{45}^W + p_{-45}^W + p_{90}^W = 1 \tag{42}$$

The evaluation function $\boldsymbol{g}$ is obtained as follows. Equation (40) is substituted into Eq. (39):

$$f = f(\boldsymbol{V}, \boldsymbol{W}) = f\left(\boldsymbol{V}_0 + a_d^V \boldsymbol{V}', \boldsymbol{W}_0 + a_d^W \boldsymbol{W}'\right)$$

$$= f_0 + f_V' + f_W' + f_{VW}' \tag{43}$$

where

$$f_0 = f(\boldsymbol{V}_0, \boldsymbol{W}_0), \qquad f_V' = a_d^V \boldsymbol{V}'^T \boldsymbol{b}_V' + \left(a_d^V\right)^2 \tfrac{1}{2} \boldsymbol{V}'^T [A_{VV}] \boldsymbol{V}'$$

$$f_W' = a_d^W \boldsymbol{W}'^T \boldsymbol{b}_W' + \left(a_d^W\right)^2 \tfrac{1}{2} \boldsymbol{W}'^T [A_{WW}] \boldsymbol{W}'$$

$$f_{VW}' = a_d^V a_d^W \boldsymbol{V}'^T [A_{VW}] \boldsymbol{W}' \tag{44}$$

$$\begin{bmatrix} \boldsymbol{b}'_V \\ \boldsymbol{b}'_W \end{bmatrix} = \begin{bmatrix} \boldsymbol{b}_V \\ \boldsymbol{b}_W \end{bmatrix} + \begin{bmatrix} A_{VV} & A_{VW} \\ A^T_{VW} & A_{WW} \end{bmatrix} \begin{bmatrix} \boldsymbol{V}_0 \\ \boldsymbol{W}_0 \end{bmatrix} \tag{45}$$

Equation (43) means that the response f can be represented by the sum of the four terms as follows: f_0 is a constant; f_V a quadratic polynomial of $\boldsymbol{V}'$; f_W a quadratic polynomial of $\boldsymbol{W}'$; and f_{VW} a linear interaction term of $\boldsymbol{V}'$ and $\boldsymbol{W}'$.

The $\boldsymbol{g}$ is defined as follows:

$$\boldsymbol{g} = f_0 + \max(f'_V) + \max(f'_W) + \max(f'_{VW}) \tag{46}$$

Assuming that $\boldsymbol{V}$ and $\boldsymbol{W}$ are independent of each other, obtain an evaluation on the outside value of $\boldsymbol{g}$. This maximization is performed within each shrunk tetrahedron region. The maximum values of f_V and f_W are easily calculated because this is a simple quadratic polynomial bounded by the tetrahedron. Because f_{VW} is the linear interaction term, the maximum value of f_{VW} locates at the apexes of the tetrahedron. The apexes of the initial tetrahedron are as follows:

$$\max f_{V'W'} = \max \left\{ \begin{array}{cccc}
f_{V'W'}\left(\begin{bmatrix}1\\1\\0\end{bmatrix},\begin{bmatrix}1\\1\\0\end{bmatrix}\right) & f_{V'W'}\left(\begin{bmatrix}1\\1\\0\end{bmatrix},\begin{bmatrix}0\\-1\\1\end{bmatrix}\right) & f_{V'W'}\left(\begin{bmatrix}1\\1\\0\end{bmatrix},\begin{bmatrix}-1\\1\\0\end{bmatrix}\right) & f_{V'W'}\left(\begin{bmatrix}1\\1\\0\end{bmatrix},\begin{bmatrix}0\\-1\\-1\end{bmatrix}\right) \\
f_{V'W'}\left(\begin{bmatrix}0\\-1\\1\end{bmatrix},\begin{bmatrix}1\\1\\0\end{bmatrix}\right) & f_{V'W'}\left(\begin{bmatrix}0\\-1\\1\end{bmatrix},\begin{bmatrix}0\\-1\\1\end{bmatrix}\right) & f_{V'W'}\left(\begin{bmatrix}0\\-1\\1\end{bmatrix},\begin{bmatrix}-1\\1\\0\end{bmatrix}\right) & f_{V'W'}\left(\begin{bmatrix}0\\-1\\1\end{bmatrix},\begin{bmatrix}0\\-1\\-1\end{bmatrix}\right) \\
f_{V'W'}\left(\begin{bmatrix}-1\\1\\0\end{bmatrix},\begin{bmatrix}1\\1\\0\end{bmatrix}\right) & f_{V'W'}\left(\begin{bmatrix}-1\\1\\0\end{bmatrix},\begin{bmatrix}0\\-1\\1\end{bmatrix}\right) & f_{V'W'}\left(\begin{bmatrix}-1\\1\\0\end{bmatrix},\begin{bmatrix}-1\\1\\0\end{bmatrix}\right) & f_{V'W'}\left(\begin{bmatrix}-1\\1\\0\end{bmatrix},\begin{bmatrix}0\\-1\\-1\end{bmatrix}\right) \\
f_{V'W'}\left(\begin{bmatrix}0\\-1\\-1\end{bmatrix},\begin{bmatrix}1\\1\\0\end{bmatrix}\right) & f_{V'W'}\left(\begin{bmatrix}0\\-1\\-1\end{bmatrix},\begin{bmatrix}0\\-1\\1\end{bmatrix}\right) & f_{V'W'}\left(\begin{bmatrix}0\\-1\\-1\end{bmatrix},\begin{bmatrix}-1\\1\\0\end{bmatrix}\right) & f_{V'W'}\left(\begin{bmatrix}0\\-1\\-1\end{bmatrix},\begin{bmatrix}0\\-1\\-1\end{bmatrix}\right)
\end{array} \right\} \tag{47}$$

Therefore, the value of Eq. (64) can be easily calculated. Because $\boldsymbol{V}$ and $\boldsymbol{W}$ are not independent variables, the actual maximum value of the response surface is surely smaller than the value of $\boldsymbol{g}$. This means that we can use Eq. (46) as an evaluation function for the branch-and-bound method.

When each laminate is evaluated, we can eliminate infeasible laminates that violate stacking-sequence constraints, such as the four-contiguous-ply rule. The stacking-sequence constraints, therefore, can be easily implemented here.

The evaluations of maximum value of Eq. (46) require only maximizations of quadratic polynomials in a shrunk tetrahedron region obtained from the fractal arrangement analysis of feasible laminates. This means the estimation can be applied for nonconvex problems. When the response surface is precise enough, this method provides global optimal laminate.

Because the branches of feasible laminates create a fractal pattern as described before, the pruning of the branch of the fractal pattern of feasible laminates to obtain optimal stacking sequence is named a fractal branch-and-bound method here. Although the mechanism of fractal pattern of the feasible laminates is used for obtaining laminates near the optimal point of the out-of-plane lamination parameters in our previous paper,[24] the mechanism of fractal pattern is not directly contributes to the pruning algorithm in the present study. For estimations of Eq. (46), the tetrahedron region, however, shrinks as the fractal pattern mechanism and each shrunk tetrahedron region are calculated on the basis of the fractal pattern mechanism. This is the reason why this method is named the fractal branch-and-bound method in the present study.

III. Optimization Problem and Results

The new fractal branch-and-bound method is applied to a buckling load maximization problem of a hat-type stiffener to confirm an effectiveness of the method. The configuration of the hat-type stiffener is shown in Fig. 6: the length (x-coordinate direction) a is 4 m, the width (y-coordinate direction) b is 3 m, the height of the hat part h is 1 m, the width of the hat part w is 1 m, and the thickness of the laminate is 2 mm (16 plies). Reference compression load $Nx = 1$ N/m is applied in the longitudinal direction as shown in Fig. 6, and the compression load is applied to the entire edge of the flange and the web. The edge of the lower flange is simply supported here. The materials used are Toray T300/5208-type unidirectional graphite/epoxy composites, and the material properties are $E_L = 181$ GPa, $E_T = 10.3$ GPa, $G_{\mathrm{LT}} = 7.17$ GPa, and $\nu_{\mathrm{LT}} = 0.28$. The fiber direction of 0-deg ply is x direction here. In this example we deal a symmetric balanced laminate. That makes $V^*_3 = 0$. Moreover, next inner angle ply to 45-deg ply is automatically set to -45-deg ply as Harrison et al.[8] introduced in the present study. This also reduces the value of W^*_3, and W^*_3 is approximately set to zero here. The total number of lamination parameters here are, therefore, four (V^*_1, V^*_2, W^*_1, and W^*_2).

A commercial base finite element method (FEM) code ANSYS is used here to calculate the buckling load of various stacking sequences selected by the D-optimal laminates listed in Table 1. Standard composite shell element (eight-node isoperimetric: shell99) is adopted, and total number of elements is approximately 2000. The FEM results obtained are listed in Table 1. These values are the buckling load ratio: the calculated buckling load (N/m) against to the reference load Nx (1 N/m). From the obtained 36 results a response surface to approximate the buckling load ratio is constructed

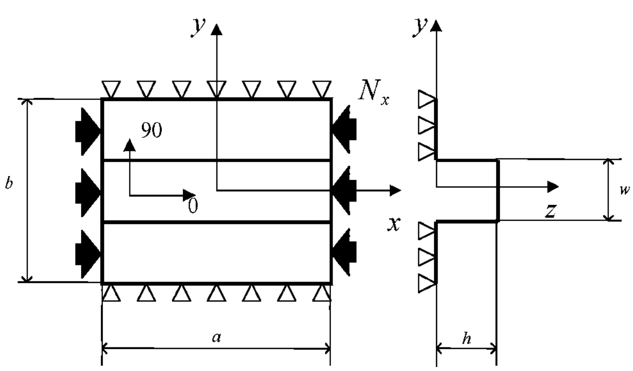


Fig. 6 Configuration of a hat-type stiffener.

Table 2 Optimal laminates obtained by the new fractal branch-and-bound method

N	Stacking sequence	f_R	f_F	Error, %
6	[45/−45/45/−45/90/90]s	3.89	3.93	−1.0
8	[45/−45/45/−45/45/−45/90/90]s	3.89	3.93	−1.0
10	[45/−45/45/−45/45/−45/45/−45/90/90]s	3.89	3.93	−1.0
12	[45/−45/45/−45/45/−45/45/−45/90/90/90/90]s	3.89	3.93	−1.0
14	[45/−45/45/−45/45/−45/45/−45/45/−45/90/90/90/90]s	3.89	3.93	−1.0

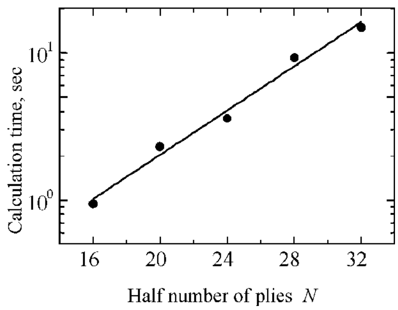


Fig. 7 CPU time of various half-number of plies.

as described in the preceding section. Because obtained response surface is as follows:

$$c = 3.1952, \qquad \begin{bmatrix} \boldsymbol{b}_V \\ \boldsymbol{b}_W \end{bmatrix} = \begin{bmatrix} -0.3168 \\ -0.2869 \\ -0.4815 \\ -0.5054 \end{bmatrix}$$

$$\begin{bmatrix} \boldsymbol{A}_{VV} & \boldsymbol{A}_{VW} \\ \boldsymbol{A}_{VW}^T & \boldsymbol{A}_{WW} \end{bmatrix} = \begin{bmatrix} 0.3190 & 0.1789 & -0.2201 & 0 \\ & -0.2128 & 0.1512 & -0.0580 \\ & & -1.3832 & -0.0722 \\ \text{Sym.} & & & 0 \end{bmatrix} \tag{48}$$

The adjusted coefficient of multiple determination R^2_{adj} (R-square-adjusted) is 0.9988. That means the approximation is excellent.

An optimal stacking sequence is obtained with the new fractal branch-and-bound method using the obtained response surface. Because the thickness of the laminate is 2 mm, the total number of stacks is normally 16 (half number of plies $N = 8$) when we use normal prepreg. To investigate the efficiency of the new fractal branch-and-bound method, we calculated several cases that have a different number of plies without changing the total thickness of the laminate. This means we virtually use thin prepreg. The results are shown in Table 2. In the table the buckling-load ratios f_R are calculated values with the response surface, and the values of f_F are calculated from FEM analyses. The calculation CPU time with a PC of Pentium II 400 MHz is shown in Fig. 7. The optimal results obtained are all true optimal stacking sequences that maximize the response surface. This optimality is confirmed with searching all feasible laminates. Error of the response surface is only −1.0% for the optimal laminate. The optimal stacking sequence is similar to the optimal stacking sequence of a maximum buckling load of a simply supported square laminate shown by Jones et al.[27]

Figure 7 shows that the CPU time is approximately proportional to the 1.2^N here. As in Harrison et al.,[8] when trinary numbers are adopted to represent the laminates, the total number of laminates is proportional to 3^N. This means that the new fractal branch-and-bound method gives excellent small computational cost.

It is true that all feasible laminates can be evaluated when the approximation of the quadratic polynomial is excellent for stacking-sequence optimizations of a laminate. This method must be applied to more complicated structures that have several laminates that need stacking-sequence optimizations. In the present study the simple structure is adopted to confirm the true optimal stacking sequence. More improvement for complicated structures is required, and this is our future work.

Because the optimality significantly depends on the accuracy of the response surface around the optimal point of lamination parameter coordinates, the new fractal branch-and-bound method requires precise approximation. How to obtain a precise response surface just around the optimal point will be a future work.

IV. Conclusions

For the stacking-sequence optimizations we proposed a new improved fractal branch-and-bound method with an approximation of a response surface in lamination parameters. The present paper describes the mathematical background of fractal pattern created with entire plots of feasible laminates in lamination parameter coordinates. After making an approximated objective function with a quadratic-polynomial response surface from the selected D-optimal laminates, fractal branches of the laminates are pruned using conservative estimation of the branch. The improved method can be applied to any complicated structures that include both in-plane and out-of-plane lamination parameters for response surface approximations of the objective functions. The proposed new method is applied to a buckling-load maximization problem of a hat-type stiffened laminate, and the method is shown to be quite efficient for stacking-sequence optimizations.

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