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Stacking sequence optimizations for composite laminates using fractal branch and bound method: Application for supersonic panel flutter problem with buckling load condition

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Abstract—The fractal branch and bound method was developed by the authors for optimization of stacking sequences to maximize buckling load of composite structures. The method demands an approximation of a design space with a response surface comprising quadratic polynomials for pruning fractal branches of stacking sequences. Approximation of the objective function with quadratic polynomials was confirmed for buckling load maximizations and flutter speed limit maximizations using lamination parameters as predictors. In the present study, flutter speed maximization with a constraint of buckling load is employed as an example of stacking sequence optimization by means of the fractal branch and bound method with a strength constraint. The present paper describes the theoretical background of the fractal branch and bound method. Then approximations are performed using quadratic polynomials with lamination parameters as predictors. After that, effectiveness of this method for supersonic panel flutter of composite laminates was investigated using two cases. Results indicate that the method was applied successfully; a practical optimal stacking sequence was obtained using modified response surfaces.

Keywords: Composites; optimization; stacking sequence; flutter; supersonic; response surface.

1. INTRODUCTION

For next-generation supersonic transportation aircraft, it is indispensable to make use of many composite structures to reduce weight and improve aero-elastic response. Composite laminates made from unidirectional prepreg sheets are employed for these composite structures because of their superior mechanical performance. However, it is required to optimize stacking sequences of the laminated

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composites. Miki [1] and Fukunaga and Chou [2] proposed a graphical stacking-sequence optimization technique with lamination parameters.

For practical laminated composite structures, available fiber angles are quite limited to a small set of fiber angles such as 0°, ±45°, and 90° plies because of the shortage of other experimental data required for design and a hand lay-up processing method. A stacking-sequence optimization problem becomes a combinatorial optimization problem with some constraints because there are several constraints on the stacking sequences.

The author's group has proposed a stacking sequence optimization method with a genetic algorithm [3–5]. A new technique to reduce evaluation cost with response surface approximations in lamination parameters was employed because the genetic algorithm requires high computational cost engendered in the necessity for a large number of evaluations of individuals [6–8]. These studies revealed that approximations of the objective function with a quadratic polynomial are applicable for buckling load maximization problems in the lamination parameters.

However, use of genetic algorithms is a probabilistic approach; it does not always give a true optimal result. This problem was solved using a newly proposed fractal branch and bound method [9]. The method requires approximation of the objective function in the lamination parameters with a quadratic polynomial for trimming branches. This method was applied to a buckling load maximization problem. Its efficiency was confirmed in our previous paper [9].

The panel flutter problem of SSTs, which is a coupled problem of fluid dynamics and structural analysis, requires larger computational cost than the buckling problem of a laminate. The panel flutter problem usually has sharp deviation of flutter speed limit, which results from changes of vibration modes [12]. This may create multiple local maxima [10]. Therefore, confirmation of the fitness of the approximation of the response surface must be done with a quadratic polynomial to employ the fractal branch and bound method on optimization of stacking sequence of the panel flutter problem. In our previous study [11], a modified response surface was proposed and the method was confirmed for the panel flutter problem in supersonic flow.

For practical design of laminated composite structures, maximizations of the flutter speed usually accompany design requirements like buckling load, strength, stiffness, etc. Our previous fractal branch and bound method did not address these constraints to optimize the stacking sequences.

The fractal branch and bound method is improved in the present study to obtain the optimal stacking sequence with a design requirement. A maximization of panel flutter speed limit is performed with a requirement of buckling load. This problem is a simple example to demonstrate the feasibility of the improved method. To confirm true optimality, we selected a simple example for application of the improved method, but application of this new improved method is not limited to simple panel structures.

2. OPTIMIZATION PROBLEM

2.1. Panel flutter analysis of laminates

Flutter speed of a simply supported rectangular ($a \times b$) laminated panel is addressed here (Fig. 1). The material properties used here are $E_L = 142$ GPa, $E_T = 10.8$ GPa, $G_{LT} = 5.49$ GPa, $\nu_{LT} = 0.3$, $\rho = 1.5 \times 10^3$ kg/m^3, respectively. The upper surface of this laminated panel is exposed in the steady supersonic flow ($\boldsymbol{V}$). When supersonic flow is steady, the flow can be approximated as a potential flow with an approximation of small oscillations from the steady flow. The basic equation of panel vibration can be represented by the following equation. In this equation, only the deviation of the flow velocity from the steady flow is considered to affect panel vibration.

$$D_{11}\frac{\partial^4 w}{\partial x^4} + 2(D_{12} + 2D_{66})\frac{\partial^4 w}{\partial x^2 \partial y^2} + D_{22}\frac{\partial^4 w}{\partial y^4} = -\rho_L h \frac{\partial^2 w}{\partial t^2} + p(x, y, t). \quad (1)$$

In that equation, $w(x, y)$, ρ_L, and h are displacement of the panel in the z-axis, density of the panel, and the panel thickness, respectively. In equation (1), D_{11}, D_{12}, D_{22}, D_{66} are elements of the bending stiffness matrix of the laminated panel. Because D_{13}, D_{23} become sufficiently small for balanced symmetric laminates made up of plies of 0°, ±45°, and 90°, they are neglected here. Small pressure deviation caused by the small deviation of the supersonic flow ($\boldsymbol{V}$) from steady flow is expressed as $p(x, y, t)$; this pressure can be expressed as the following.

$$p(x, y, t) = -\left(\lambda \frac{\partial w}{\partial x}\cos\varphi + \lambda \frac{\partial w}{\partial y}\sin\varphi + \mu \frac{\partial w}{\partial t}\right). \quad (2)$$

In equation (2), φ is the incidence angle to the laminate of a supersonic flow; λ and μ represent the dynamic pressure parameter and aerodynamic damping coefficient, respectively.

$$\lambda = -\frac{2q}{\sqrt{M^2 - 1}}, \qquad \mu = \frac{\lambda(M^2 - 2)}{V(M^2 - 1)}. \quad (3)$$

In equation (3), q and M are dynamic pressure of the steady flow and its Mach number, respectively.

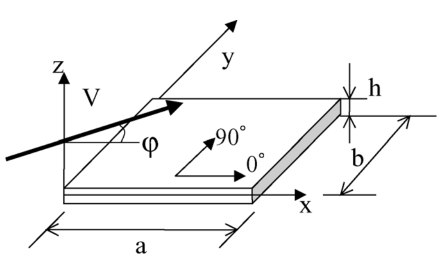

Figure 1. Coordinates of the laminate and supersonic flow.

Displacement of the simply supported rectangular composite panel can be approximated using trigonometric functions such as

$$w(x, y, t) = \sum_{m=1}^{M} \sum_{n=1}^{N} A_{mn} e^{\omega t} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}, \quad (4)$$

where ω is the frequency of a laminate and m (n) is a vibration mode.

Substituting equation (4) into equation (1) produces a complex eigenvalue problem as

$$\{[\mathbf{K}] + \lambda[\mathbf{K_A}]\}\{\mathrm{A}\} = \kappa[\mathbf{M_0}]\{\mathrm{A}\}, \quad (5)$$

where $[\mathbf{K}]$, $[\mathbf{K_A}]$ and $[\mathbf{M_0}]$ are a stiffness matrix, an aerodynamic matrix, and a mass matrix, respectively.

Eigenvalues are calculated by changing the Mach number M, i.e. λ; thereby, the imaginary and real parts of the eigenvalues are obtained. Flutter limit λ_c is the point at which the two different eigenvalue modes coincide with each other first. The obtained flutter limit is normalized as

$$\lambda_c^* = \frac{a^3}{D_0} \lambda_c, \quad (6)$$

where D_0 is the bending stiffness D_{11} when a laminate comprises only 0°-plies.

Change of stacking sequence of the laminate induces change of bending stiffness in e quation (1): that causes change of flutter limit λ_c^*.

2.2. Buckling analysis of laminate

The buckling load of a laminated panel can be calculated with an analytical formula. In the case of applied compression load shown in Fig. 1, the analytical formula is given as [14]:

$$N_{cr} = \left(\frac{\pi n_b}{b}\right)^2 \left\{ \left(\frac{m_b b}{b_b a}\right)^4 D_{11} + 2(D_{12} + 2D_{66}) \left(\frac{m_b b}{n_b a}\right) + D_{22} \right\}, \quad (7)$$

where m_b and n_b are buckling modes of the laminate, respectively.

In strict analysis, the compression load applied to the composite panel causes change of the vibration formula. The true flutter limit, therefore, differs from the result obtained from equation (1), which is obtained without applied load. Including the coupling between buckling and the flutter limit creates a more complicated problem. This problem is addressed without coupling between compression load and flutter limit because the objective of the present study is to confirm the new method of a design requirement.

3. OPTIMIZATION METHOD

3.1. Flow of optimizations

First, a response surface must be created to approximate the entire feasible lamination parameter (W_1^*, W_2^*) space before stacking sequence optimizations. In the present study, an objective function is the flutter limit λ_c^* with the requirement of a buckling load. The buckling load N_{cr} comprises only bending stiffness of a laminate as shown in equation (1). In this optimization problem, therefore, only bending lamination parameters are regarded as variables [3]. Bending lamination parameters W_3^* and W_4^* are neglected here because the number of angle plies is balanced; also, a +45-ply is placed near the −45-ply here in the same way as mentioned in reference [5]. To produce a precise approximation of the objective function with a response surface, design of experiments is performed from the candidate of entire set of feasible laminates with respect to bending lamination parameters W_1^* and W_2^*. The design process of the experiments is described in reference [6]. After selection of laminates for analyses, analyses of the panel flutter limit are conducted. Then the calculated responses (flutter limit) of all selected laminates are used to obtain coefficients of the response surface (quadratic polynomials) using the least squares error method. This procedure is also listed in reference [6].

In the present study, the design requirement is buckling load. In the next step, a response surface of buckling load with quadratic polynomials is created to deal with the design requirement. For this response surface, the response is buckling load ratio N_{cr} and variables are the bending lamination parameters (W_1^*, W_2^*). The identical set of laminates selected by the design of experiments is employed again. Buckling load ratios of the set of laminates are all calculated. Then an approximated function of buckling load ratio is obtained again with the least square errors method. The design requirement of the buckling load can be represented as a quadratic curve in the bending lamination parameter (W_1^*, W_2^*) space using this response surface of the buckling load ratio. For example, when the buckling load requirement is given, a boundary of the laminates that satisfy the requirement can be derived from the quadratic polynomials of the buckling load approximation; this must be a quadratic curve.

After obtaining the response surfaces, the optimal laminate is sought using the fractal branch and bound method using the obtained response surface of flutter limit λ_c^*, where the quadratic curve obtained from the response surface of buckling load is considered as one boundary of feasible candidate laminates.

3.2. Bending lamination parameters

A symmetric laminate of total number of plies $2Ns$ is considered here. The bending lamination parameters (W_1^*, W_2^*) of a laminate is defined as

$$\boldsymbol{W} = \begin{bmatrix} W_1^* \\ W_2^* \end{bmatrix} = \frac{3}{t^3 Ns^3} \sum_{k=1}^{Ns} \left\{ (Ns - k + 1)^3 - (Ns - k)^3 \right\} \begin{bmatrix} \cos 2\theta_k \\ \cos 4\theta_k \end{bmatrix}, \tag{8}$$

where k is the ply number counted from the outermost ply, θ_k is fiber angle of the kth ply, and t is the total thickness of the laminate. Using the lamination parameters, bending stiffness elements are expressed simply as

$$\begin{bmatrix} D_{11} \\ D_{22} \\ D_{12} \\ D_{66} \end{bmatrix} = \frac{h^3}{12} \begin{bmatrix} U_1 & W_1^* & W_2^* \\ U_1 & -W_1^* & W_2^* \\ U_4 & 0 & -W_2^* \\ U_5 & 0 & -W_2^* \end{bmatrix} \begin{bmatrix} 1 \\ U_2 \\ U_3 \end{bmatrix}, \tag{9}$$

where U_i $(i = 1\text{–}5)$ are material invariants described in reference [15].

3.3. Design of experiments and response surface method

In the present study, both flutter limit (λ_c^*) and buckling load ratio (N_{cr}) are approximated with two response surfaces. In both response surfaces, variables are identical bending lamination parameters (W_1^*, W_2^*). When the response is represented as y and variables W_1^* and W_2^* are replaced with x_1 and x_2 for simplification, a response surface of quadratic polynomials can be rewritten as

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1^2 + \beta_4 x_2^2 + \beta_5 x_1 x_2. \tag{10}$$

The six unknown coefficients are calculated with the least square error method from 12 sets of calculated responses and the lamination parameters. In our previous study [9], the response surface obtained from the 12 sets provides sufficiently precise approximation for the buckling load problem. On the other hand, as mentioned before, the panel flutter problem usually has sharp deviation of the flutter speed limit, which results from changes of vibration modes. This may create multiple local maxima. Our previous paper [12] proposed a modified global response surface for approximation of the panel flutter. In the present study, the modified response surface is employed to approximate the panel flutter limit. In the modified response surface method, 15 sets instead of 12 sets are used for the least square error method.

The extra sets are not newly selected laminates. In the first step of this method, the maximum laminate in the response values of the 12 sets is obtained from the 12 sets. The maximum laminate of the 12 sets is added as extra three times to the data sets for regression; then a response surface is recreated from the 15 $(= 12 + 3)$ laminates. Although the method uses 15 sets, the method requires only 12 sets for calculation. This method reduces the bias just around the provisional optimal point. In addition, this method is very effective for approximation of flutter limit, which has a rapid change of the response caused by the mode change.

Design of experiments with D-optimality is performed to reduce variance of the coefficients; 12 sets of the laminates are selected as shown in Fig. 9 from candidates of feasible laminates. Because the available fiber angles are limited to the small set of 0°, ±45° and 90°, the feasible space of the bending lamination parameter is bounded by the triangular domain of the three apexes, $(W_1^*, W_2^*) = (-1, 1),\ (1, 1),\ (0, -1)$. The reason is described in detail in references [6–8].

3.4. Fractal branch and bound method

Bending lamination parameters are expressed as progressions by means of counting ply location from the outermost ply of a laminate as shown in equation (8). Because available fiber angles are limited to a 0°-ply, a ±45°-ply, and a 90°-ply, the values of $(\cos 2\theta, \cos 4\theta)$ in the equation (8) are also limited to $0° = (1, 1)$, $\pm 45° = (0, -1)$, and $90° = (-1, 1)$. These values are identical to the vertex vectors of the triangle area of the bending lamination parameters (W_1^*, W_2^*) made from feasible laminates. All of the coefficients of vectors of equation (8) are real numbers from 0 to 1 and bending lamination parameters of any stacking sequences are the linear sum of these three vectors. Thus, when all feasible laminates are plotted on the bending lamination parameters, a fractal pattern is created inside and on the boundary of the triangle. The fractal pattern of half-ply number $Ns = 6$ (a symmetric laminate of total 12 plies) is shown in Fig. 2 as an example.

Figure 2 shows an entire set of laminates that have a 0°-ply in the outermost ply is plotted in the shrunken triangle of the upper right. Correspondingly, an entire set of laminates that have a 90°-ply in the outermost ply is plotted in the shrunken triangle of upper left; an entire set of laminates that have a 45°-ply in the outermost ply is plotted in the shrunken lower triangle. When the second ply counted from outermost ply is decided, each shrunken triangle has other more shrunken triangles inside of each triangle. In this manner, with the decision of ply angles toward inside plies, this self-similar shrunken triangle is repeated inside the each triangle and the entire fractal pattern is formed. Each triangle domain of laminates is called a fractal branch in the present study.

A provisional optimal point W_{op}^* is easily obtained when we assume the lamination parameters are continuous real numbers using the response surface. For feasible laminates, however, available fiber angles are limited to a small set of fiber angles. This limitation makes the practical lamination parameters discrete numbers. The discrete number of practical lamination parameters causes a problem: a laminate

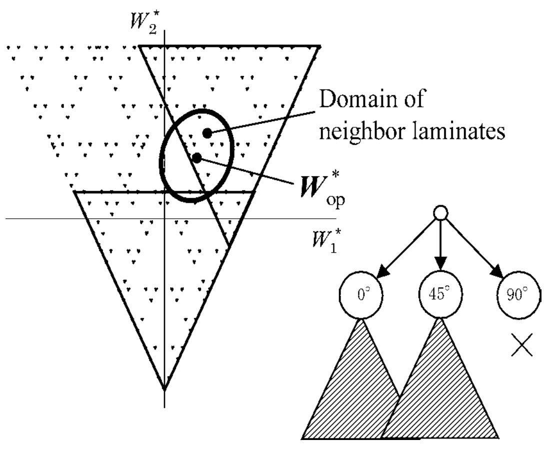

Figure 2. Fractal structure of design space.

that has exactly the same lamination parameters as the point of W^*_{op} does not always exist practically.

The branch and bound method for optimizing stacking sequence is performed to solve this problem. As previously mentioned, the provisional optimal point of W^*_{op} is obtained first, all feasible laminates located within the small domain near the provisional optimal point are collected. All feasible laminates in the domain around the provisional optimal are evaluated with the objective function of the response surface. The optimal stacking sequence that has the maximum value of the objective function is selected from the collection.

Fractal braches are used in the present study to collect the laminates around the provisional optimal point efficiently. The method is shown schematically in Fig. 2. In this figure, two shrunken triangles (fractal branches) intersect the domain around the provisional optimal point. Because the branches that have a 90^{o}-ply in the outermost ply do not intersect with the domain around the provisional optimal point as shown in Fig. 2, the fractal branch (a set of laminates) does not have the optimal laminate. This allows pruning of the fractal branch. The lower hierarchies of the fractal branches are pruned similarly. In this way, all feasible laminates within the domain near the provisional optimal point W^*_{op} are obtained. After obtaining the set of candidates around the optimal point W^*_{op}, the actual optimal stacking sequence can be selected by means of estimations of all candidate laminates using the response surface. This branch and bound method for optimizing the stacking sequence of a laminate is named a fractal branch and bound method in the previous study [9]. Detail description of the fractal branch and bound method is in reference [9].

The fractal branch and bound method is improved in the present study to obtain the optimal stacking sequence with a design requirement as described previously. Maximization of the flutter limit with a design requirement of buckling load is performed here. The response surface of buckling load is used to consider the design requirement of buckling load. As previously mentioned, when the buckling load requirement is given, a boundary of the laminates that satisfy the design requirement can be solved from the quadratic polynomials of the buckling load approximation; this is a quadratic curve. After obtaining the curve, the optimal stacking-sequence is searched using the fractal branch and bound method. On the search using the obtained response surface of flutter limit λ^*_c, the quadratic curve of the design requirement of buckling load is considered as one boundary of feasible candidate laminates. The new method has three categorized patterns depending on where the provisional maximum point of a response surface of flutter limit λ^*_c exists. Each pattern is described in detail as follows.

Pattern 1: The maximum point of the response surface of objective function (flutter limit) exists inside of the design space bounded by the design requirement. The point is regarded as a provisional optimal laminate. One of the boundaries of the design space is a quadratic curve obtained from the quadratic polynomial response surface of the design requirement (buckling load) (see Fig. 3). In this case,

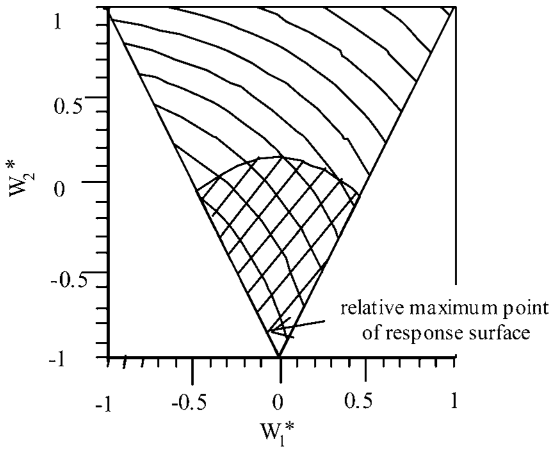

Figure 3. Contour plot of response surface of flutter speed limit with limitation of buckling load ratio (Pattern 1).

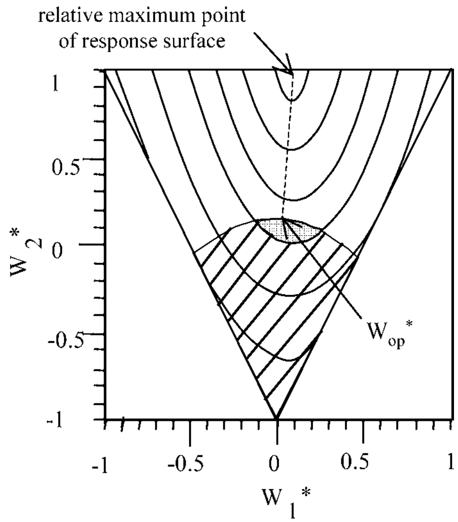

Figure 4. Contour plot of response surface of flutter speed limit with limitation of buckling load ratio (Pattern 2).

the fractal branch and bound method is normally performed in the design space with the design requirement. The resultant optimal laminate has a maximum flutter limit satisfying the requirement of buckling load.

Pattern 2: The maximum point of the response surface of objective function (flutter limit) exists outside of a design space bounded by the design requirement (buckling load) (see Fig. 4). In this case, the provisional optimal point of the response surface of objective function (flutter limit) inside of the design space must exist on the boundary of the design space with the design requirement. As described before, the boundary of design space with the design requirement is a quadratic

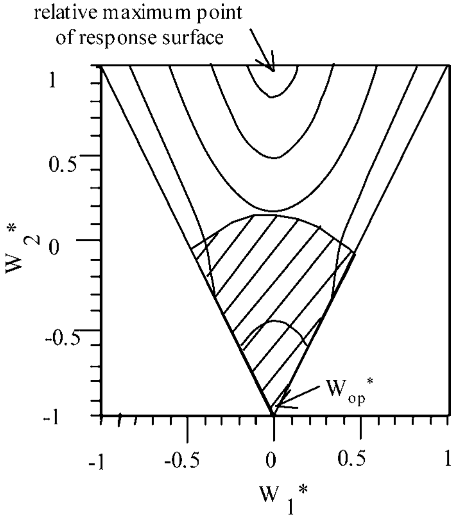

Figure 5. Contour plot of response surface of flutter speed limit with limitation of buckling load ratio (Pattern 3a).

curve. The maximum point of the objective function (flutter limit) on the boundary with the quadratic curve is adopted for the provisional optimal point W^*_{op}. Using this provisional optimal point W^*_{op}, the fractal branch and bound method is performed in the design space with design requirement. The obtained optimal laminate has the maximum flutter limit and satisfies the buckling load requirement.

Pattern 3: The response surface of the objective function (flutter limit) has two local maxima. The response surface makes a saddle form because the quadratic polynomials are adopted here. When the larger maximum point of the local maxima exists inside the design space with the design requirement (see Fig. 5), the maximum point is regarded as a provisional optimal point and the optimal laminate maximizing objective function (flutter limit) with the design requirement (buckling load) is obtained in the same method as described in pattern #1.

When the smaller local maximum point exists inside the design space with the design requirement (see Fig. 6), the maximum point of the response surface of objective function should be the smaller local maximum or the maximum point of the response surface of objective function on the boundary with the quadratic curve (see Fig. 7). The larger point is adopted as the provisional optimal point W^*_{op}. Using this provisional optimal point W^*_{op}, the fractal branch and bound method is performed in the design space with the design requirement. The obtained optimal laminate has the maximum flutter limit with the buckling load requirement.

Because the design space of the flutter problems must be a convex form or a saddle form, by means of the three techniques, the true optimal laminate that maximizes flutter limit with the requirement of buckling load is obtained using the fractal branch and bound method.

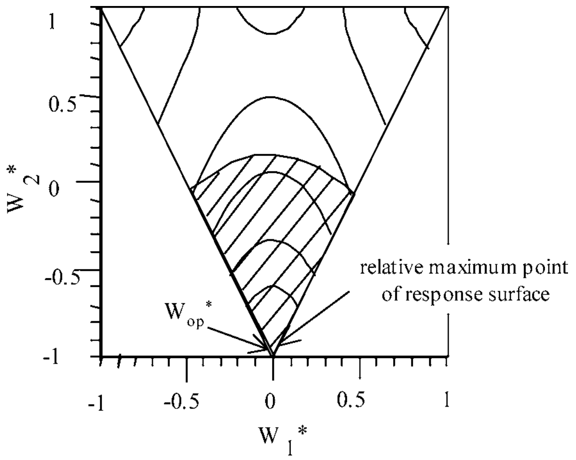

Figure 6. Contour plot of response surface of flutter speed limit with limitation of buckling load ratio (Pattern 3b).

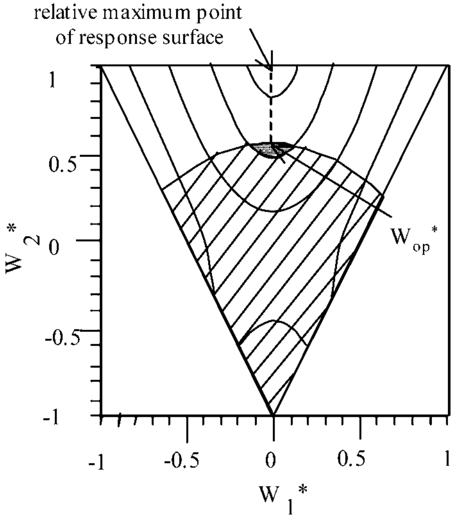

Figure 7. Contour plot of response surface of flutter speed limit with limitation of buckling load ratio (Pattern 3c).

4. RESULTS OF OPTIMIZATION AND DISCUSSION

The new method is adopted in this study to two stacking-sequence optimization problems maximizing the flutter limit with the buckling load requirement. One problem is optimization of a square laminate and the other is optimization of a rectangular laminate.

4.1. Optimization results for the square laminate

An optimization model is a square laminate of $a = b = 1$ [m] as shown in Fig. 1. The stacking sequence of the laminate is optimized with a requirement of buckling load using the fractal branch and bound method.

As described previously, first a response surface of flutter limit must be created to approximate the entire feasible lamination parameter space (W_1^*, W_2^*). Design of experiments with D-optimality is performed to make a precise approximation of the objective function with a response surface; thereupon, 12 sets of laminates are selected, as shown in Fig. 8, from the candidates of feasible laminates. However, the origin point is surely included in the D-optimality design of the experiment to reduce bias around the origin point.

Flutter analyses are performed in all 12 selected laminates. In flutter analyses, the displacement function in equation (4) is determined to include all vibration modes up to the fourth mode. In the present problem, the design requirements of buckling load are set to be more than $N_{cr} = 3.5 \times 10^5$ N/m. Using flutter analysis results obtained for the 12 laminates, a response surface is created with the modified response surface method: 15 data are used for making the response surface. Figure 9 shows the contour plot of the obtained response surface.

First, a stacking-sequence optimization maximizing flutter limit with the normal fractal branch and bound method is performed without considering the buckling load requirement. The obtained optimal laminate is $[(0/90)_4]_s$. The coordinate in the bending lamination parameters space of this provisional optimal laminate is $(W_1^*, W_2^*) = (0.25, 1)$. Laminates adjacent to the provisional optimal laminate are analyzed, and optimality is confirmed. The value of the flutter speed limit λ_c^* is 116.0 for the provisional optimal laminate.

Next, a response surface of buckling load ratio is created. Figure 10 shows the contour plot of the response surface obtained. The design space with the requirement of buckling load is shown as a slashed area in the figure. The contour

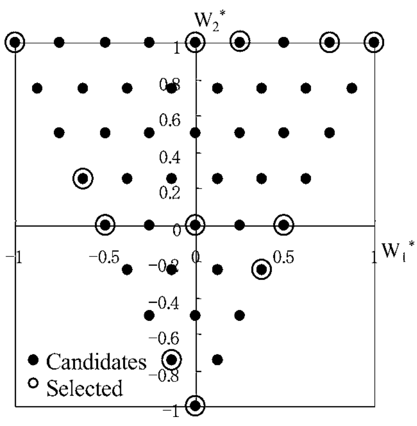

Figure 8. Candidate points and selected points.

plot of the response surface of the flutter limit (Fig. 9) is superimposed on the response surface of buckling load (Fig. 10) as shown in Fig. 11. As shown in this figure, this problem is classified to the pattern 3 and the local maximum point is included inside of the design space.

In this design space with the design requirement of buckling load of the slashed area in Fig. 11, the modified fractal branch and bound method described in a preceding section is performed. The modified method yields the optimal laminate that maximizes the flutter limit with the requirement of buckling load. The obtained optimal stacking sequence is $[45/-45/45/-45/45/0_2/-45]_s$. The coordinate in the bending lamination parameters space of this laminate is $(W_1^*, W_2^*) = (0.051, -0.898)$, and the value of λ_c^* is 115.4. The laminates adjacent to the optimal laminate are analyzed. The true optimal laminate is $[45/-45/45/0/-45/0_2/90]_s$, and the coordinate in the bending lamination parameters space is $(W_1^*, W_2^*) = (0.168, -0.656)$;

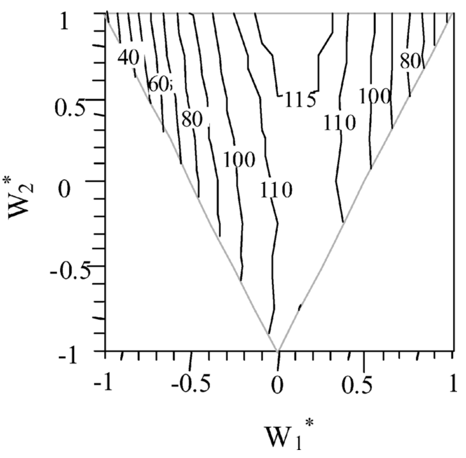

Figure 9. Contour plot of response surface of flutter speed limit $\varphi = 30° \ a/b = 1$.

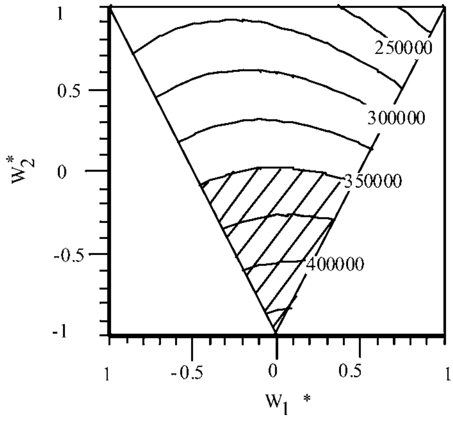

Figure 10. Contour plot of response surface of buckling load ratio $a/b = 1$.

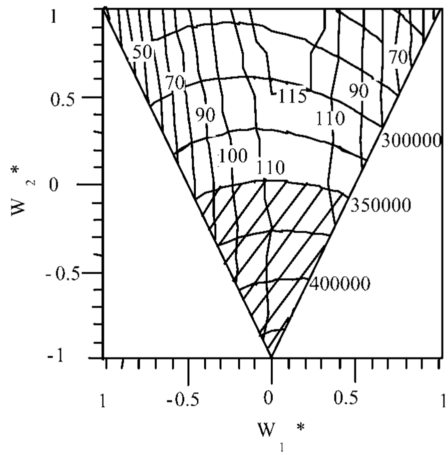

Figure 11. Contour plot of response surface of flutter speed limit with buckling load ratio $\varphi = 30°$ $a/b = 1$.

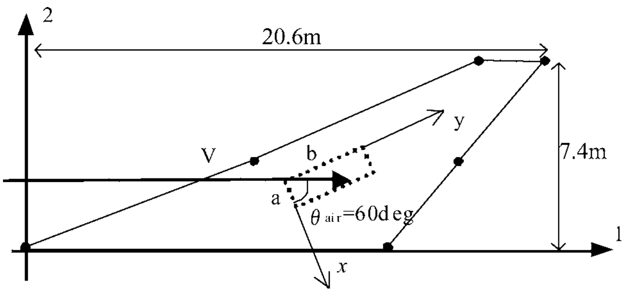

Figure 12. Coordinates of SST wing and supersonic flow.

the value of λ_c^* is 116.4. The error of the flutter speed limit against the true optimal laminate is less than 1%. It is confirmed that the obtained optimal laminate is acceptable practically.

4.2. Optimization result of rectangular laminate

The optimization problem for a rectangular laminate is performed as an example for supersonic transportation aircraft (see Fig. 12). The optimization target is a rectangular laminate with an aspect ratio of $a/b = 1/3$. The coordinates of 12 laminates for making a response surface are the same as the previous problem (see Fig. 9). The design requirement of the buckling load is set to be more than $N_{cr} = 1.0 \times 10^7$ N/m in this optimization problem. As with the previous optimization problem, the flutter analyses are performed for the selected 12 laminates. In addition, a response surface of flutter limit is made using the modified response surface method. The contour plot of the obtained response surface is shown in Fig. 13.

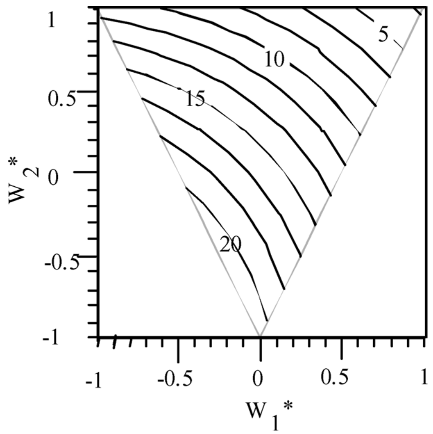

Figure 13. Contour plot of response surface of flutter speed limit $\varphi = 60^\circ$ $a/b = 1/3$.

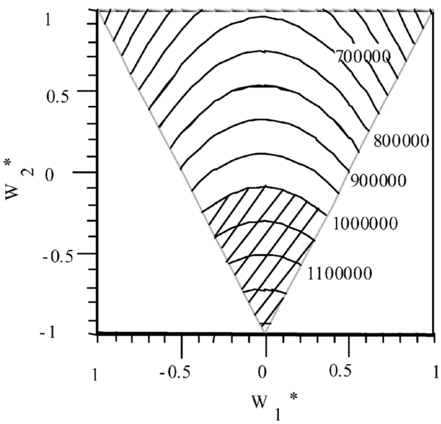

Figure 14. Contour plot of response surface with buckling load ratio $a/b = 1/3$.

First, a stacking-sequence optimization that maximizes the flutter limit using the normal fractal branch and bound method is performed without considering the design requirement of buckling load. The obtained optimal laminate is $[45/{-45}/45/90/{-45}/90_3]_s$. The coordinate of the bending lamination parameter of this laminate is $(W_1^*, W_2^*) = (-0.172, -0.656)$ and the value of λ_c^* of this laminate is 20.4. Laminates adjacent to the optimal laminate are analyzed to confirm optimality of the obtained optimal laminate. The true optimal laminate is $[(45/{-45})_4]_s$, the coordinate in the bending lamination parameters space is $(W_1^*, W_2^*) = (0.0, -1.0)$ and the value of λ_c^* is 20.9. The error of the flutter speed limit against the true optimal laminate is less than 2.4%. The obtained optimal laminate is acceptable practically.

Next, the response surface of the buckling load ratio is created identically to the previous problem. Figure 14 shows the obtained response surface of the buckling load. The design space with the requirement of buckling load is shown as the slashed area in the figure. The response surface of flutter limit (see Fig. 13) is superimposed to the response surface of the buckling load (see Fig. 14) as shown in Fig. 15. As shown in this figure, this problem is pattern #1. The provisional optimal solution is the previously obtained laminate without considering the design requirement of buckling load. For this problem shown in Fig. 15, the modified fractal branch and bound method is performed as described previously. The optimal laminate that maximizes the flutter limit with the requirement of buckling load is obtained. The obtained optimal laminate is $[45/-45/45/90/-45/90_3]_s$. The coordinate in the bending lamination parameters space of this laminate is $(W_1^*, W_2^*) = (-0.172, -0.656)$, and the value of λ_c^* is 20.4. It is confirmed that the optimal laminate obtained is acceptable practically.

The flutter limit of the optimal laminate is obtained from the approximation with the response surface. Therefore, for actual designs of laminated composite structures, additional flutter analysis must be carried out again for the obtained optimal laminate. Thus, the approximated coordinate of the maximum point in the bending lamination parameter space is more important than the approximated flutter limit of the optimal laminate. Because the proposed method provides a very accurate coordinate of optimal laminate, it is confirmed that the response surface approximation with quadratic polynomials is very effective even for the design space of flutter limit with the requirement of buckling load. As previously mentioned, to perform the fractal branch and bound method for stacking-sequence optimization of laminates, the objective function must be approximated precisely with a response surface made from quadratic polynomials.

This study addressed flutter speed maximizations with the buckling load requirement. However, this method is not limited to flutter speed maximizations, and this method can be applied to any kind of problems whose objective function can be approximated with a response surface made from quadratic polynomials. The design requirement itself is also not limited to the buckling load.

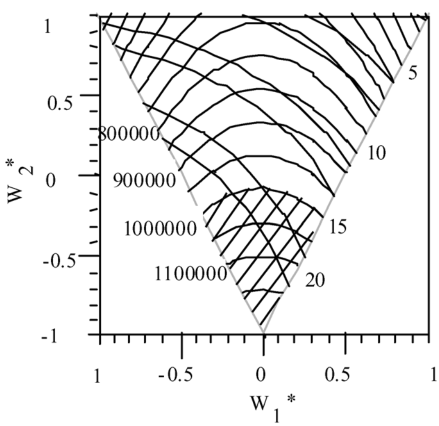

Figure 15. Contour plot of response surface of flutter speed limit with buckling load ratio $\varphi = 60°$ $a/b = 1/3$.

5. CONCLUSIONS

The fractal branch and bound method was applied to stacking-sequence optimization of composite laminates. To perform the fractal branch and bound method for stacking-sequence optimizations of laminated composites, the objective function must be approximated precisely with a quadratic-polynomial response surface in the bending lamination parameter space.

In this study, the fractal branch and bound method was applied to the stacking-sequence optimization problem to maximize the flutter limit with the requirement of buckling load. A practical optimal stacking sequence was obtained. In addition, the availability of this method was confirmed. The conclusions obtained are the following:

(1) In the maximizing flutter limit problem with the requirement of buckling load, the buckling load is also approximated with a response surface made from quadratic polynomials. This approximation of the design requirement provides the boundary requirement of buckling load as a quadratic curve on the bending lamination parameters space.

(2) In stacking sequence optimization for maximizing the flutter limit with the requirement of buckling load, the flutter limit design space can be approximated precisely using the modified response surface method; the optimal stacking-sequence is obtained using the fractal branch and bound method. The proposed method requires only a low computational coast.

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