Application of Genetic Algorithms to Stiffness Optimization of Laminated Composite Plates with Stress-Concentrated Open Holes*

Akira TODOROKI**, Kyu WATANABE***
and Hideo KOBAYASHI**

Recently, laminated composite plates have been applied to many aircraft structures because their mechanical properties are superior to those of conventional materials. Since the laminates have anisotropic elastic properties, an optimum design is needed to make advantageous use of composite laminates. The authors have proposed a successful object-oriented expert system to design laminated composites with actual constraints. A solution to optimize the fiber orientation of stiffeners around open holes, however, has not yet been developed because it is very difficult and requires stress redistribution analysis. In this study, therefore, genetic algorithms (GA) were applied to solve the optimization problem by the object-oriented finite-element stress analysis method. Results obtained were as follows. (1) The GA method is applicable to an open hole model. (2) The random search method is easily applicable to any model.

Key Words: Composites, Laminates, Genetic Algorithm, Optimum Design, Stress Concentration

1. Introduction

Because of its excellent specific stiffness and strength, a laminated composite plate, which is made by stacking unidirectional prepreg in various fiber directions, is used for primary structure of aerospace vehicles. Optimum material design is necessary, however, so that the laminates will have anisotropic properties.

Several authors have already developed an optimum design expert system for the laminated composite plate[(1)]. However, it is still necessary to develop an optimization system which is combined with a stress analysis system, in order to analyze complex structure.

We have developed an object-oriented finite-element analysis system and examined an object-oriented optimization method for the problem of optimizing the fiber orientation of stiffeners around an open hole and have shown its effectiveness[(2)].

In the previous papers, a combinatorial optimization problem with fiber orientation of stiffeners selected from limited angles to reduce concentration of strain energy was dealt with. Recently, genetic algorithms, based on random numbers, are attracting attention because of their efficiency insolving combinatorial optimization problems[(3)-(6)]. In particular, Riche and Haftka applied genetic algorithms to the optimization problem of laminated composite plates and reported a good result[(6)].

In this study, therefore, the genetic algorithms were applied to the bolt model, which could not be sufficiently optimized by the object-oriented optimization system, as described in the previous paper, and their effectiveness was examined.

Furthermore, improved genetic algorithm was developed and compared with a conventional genetic algorithm.

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** Department of Mechano-Aerospace Engineering, Tokyo Institute of Technology, 2-12-1 O-okayama, Meguro-ku, Tokyo 152, Japan
*** Graduate Student of Tokyo Institute of Technology, 2-12-1 O-okayama, Meguro-ku, Tokyo 152, Japan

2. Application of Genetic Algorithm to Optimization Problem

2.1 Optimization problem

In this study, the optimization problem involves attaching stiffeners around a stress concentrated region in unidirectional CFRP laminated composite plates, and reducing the strain energy concentration.

The details of the problem and constraint conditions are listed below.

(1) The laminated composite plate is quasi-isotropic and the stacking sequence of the plate is unchanged. The material is CFRP (T300/5208 : E_L=181 GPa, E_T=10.3 GPa, G_{LT}=7.2 GPa, ν_{LT}=0.28).

(2) Fiber orientation of stiffener is limited to 0 degrees ([0_2]), 90 degrees ([90_2]) and 45 degrees ([45/-45]), because it is assumed that stiffeners are attached by hand.

(3) Since the stiffeners are attached after curing of the laminated composite plate, different kinds of stiffener can be used on adjoining elements or element groups in this study.

(4) There are two possible attachment methods : to attach a sheet of stiffener or nothing on either individual elements, or element groups.

(5) Forced displacement is applied. Since the amount of applied force increases due to the addition of stiffener, the forced displacement is reduced until the amount of applied force becomes equal to the initial value.

(6) If the largest strain energy in all elements is the most largely reduced, the distribution of stiffener is regarded to be the best solution.

(7) The models used for optimization are hole models, which is 12 mm wide, and 1 mm thick with a 4-mm-diameter hole (shown Fig. 1), and a bolt model (shown Fig. 2). In the bolt model, it is assumed that a bolt is inserted into the hole. One-quarter of the hole model structure and one-half of the bolt model structure are analyzed because of their symmetry.

It is necessary, in the bolt model, to consider that the bolt comes into contact with the edge of the hole after the hole deforms. In this study, however, to simplify the problem, the nodes, which are located at the right hole edge, are constrained to simulate bolt insertion. The condition is the same as in case of inserting a thin bolt in the hole.

In the hole model, there is tensile stress concentration at the edge of the hole. When stiffener of 0 degrees is attached to the elements at the specimen edge, it is clear that the load around the hole is decreased and stress concentration is reduced. Conversely, in the bolt model, compression stress is produced on the right side of the hole and stress distribution is comparatively more complex than the case of the hole model. The optimization by the object-oriented optimization method was therefore not effective because it depends on the order of the element number[2].

2.2 Application of genetic algorithms

The genetic algorithms are applied to the problem above.

The details of the genetic algorithms are omitted because they have already been described in the literatures[3]-[6]. An outline of the genetic algorithms is shown in Fig. 3.

A chromosome, which consists of combined genes, represents a solution. Therefore a set of chromosomes (individuals) represents a solution set. The superiority of each individual is evaluated and individuals are selected according to their superiority, to make a new set of individuals. Crossover and mutation occur in selected individuals with a fixed probability. The next-generation set is produced by these operations. Evaluation, selection, crossover and mutation are repeated and further new generations of individuals are produced. The analysis is stopped when the best superiority become constant for some generations.

Factors such as pulation size, probability of selection, mutation, and crossover, are selected by parameter survey which is applied to the hole model, because the solution is already available ; the bolt model was analyzed using these parameters.

The following setting was performed to apply

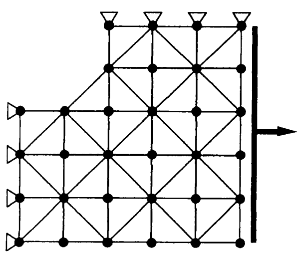

Fig. 1 The hole model

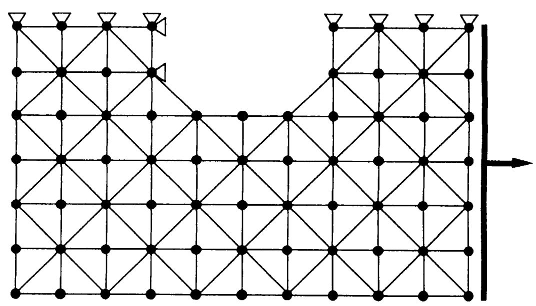

Fig. 2 The volt model

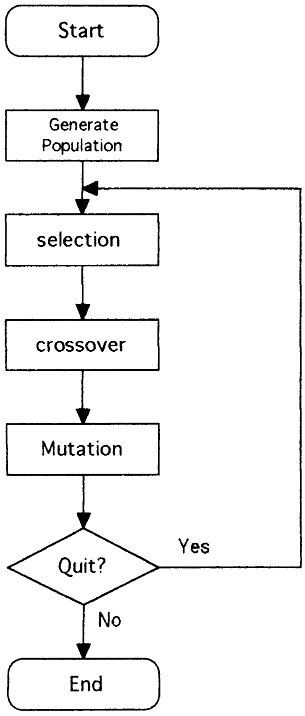

Fig. 3 Flow chart of genetic algorithm

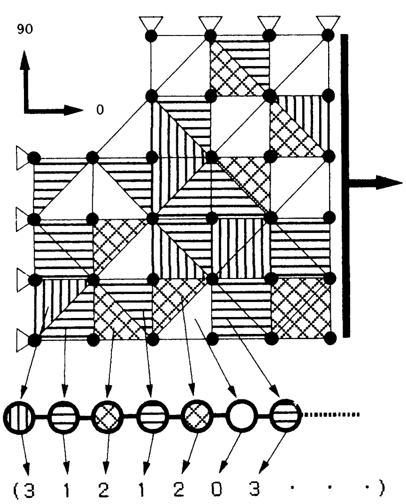

Fig. 4 The relationship between fiber orientation and gene

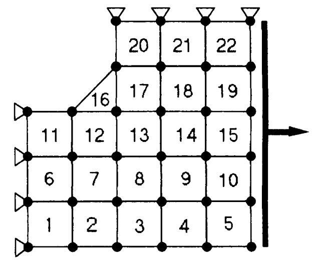

Fig. 5 (a) Grouped hole model

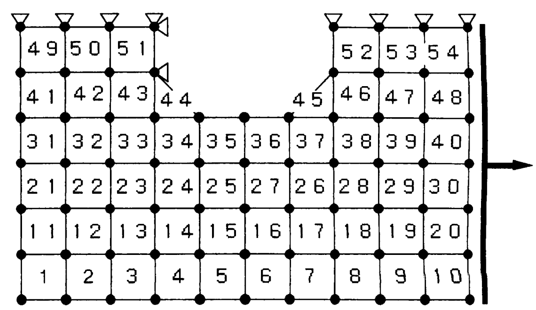

Fig. 5 (b) Grouped volt model

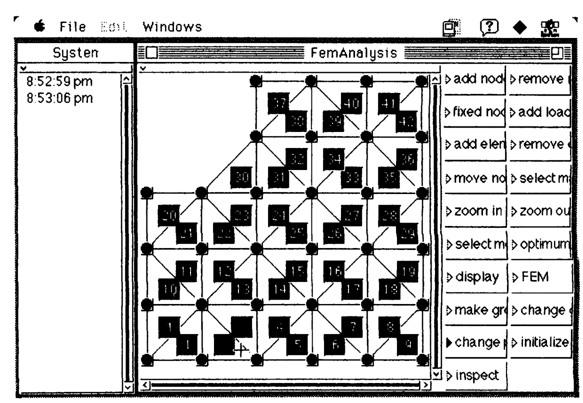

Fig. 6 User interface

genetic algorithms to optimum design of laminated composite material.

(1) Implementation of genetic algorithms

One gene corresponds to an element or to an element group and a chromosome represents the distribution of stiffener. Since 4 kinds of stiffeners, $[0]_2$, $[45/-45]$, $[90]_2$ or [nothing], are used, the figures of 0, 1, 2 and 3 were used as gene codes. The relationship between stiffener and gene is shown in Fig. 4. If a gene corresponds directly to an element, not only is the chromosome long, but also the combination number is enormous. Moreover, the angle of stiffener is not continuous. It is not necessary to be continuous, but the stiffener, whose angle differs from that of the surrounding stiffener, is undesirable from the production viewpoint. For this reason, in this study, two elements are grouped as shown in Figs. 5 (a) and (b) and the same kind of stiffener is attached to the two grouped elements. By means of a user interface based on the object-oriented finite-element system, the element can easily be grouped as shown in Fig. 6.

(2) Producing the first generation

A random number is used to decide the angle of the first-generation stiffener.

(3) Evaluation of superiority

To begin with, strain energy of each element is calculated by finite-element analysis. Then the maximum strain energy of the structure u_{max}, is settled. f is defined as u_{max}/u_{max0}, where u_{max0} is the maximum strain energy of the structure without stiffener. It is not necessary to use this f directly as an indicator of superiority of the genetic algorithms. In this study, f', which is expanded by a power function, shown as expression (1), is used for superiority in order to obtain excellent individuals for the next generation with high probability.

$$f'=f^k \quad (1)$$

(4) Selection

The roulette model is used for the selection. In the roulette model, the area on the roulette wheel allotted to each individual corresponds to its superiority, and a random number is used for selection. If it is assumed that the total area of the roulette is 1, the area of each individual S_i is set up according to the following expression using each superiority f'_i.

$$S_i=1-(f'_i/\Sigma f') \quad (2)$$

In order to prevent the best individual in a generation from being destroyed by mutation or crossover, the best individual always remains into the next generation, which is called the elite preservation strategy.

(5) Crossover

One-point crossover is adopted in this study. A pair of chromosomes is selected from the group of individuals and crossover is conducted at the center of the chromosome. The crossover is carried out with crossover probability P_c.

(6) Mutation

Mutation is carried out with mutation probability P_m. Two genes, which are mutated, are selected by a random number.

Optimum design by means of genetic algorithms was conducted based on the above analysis procedures.

Genetic algorithms fundamentally depend on a random number. Even for a rough search using a simple algorithm, an approximate solution is expected to be obtained. The calculation efficiency, however, differs greatly depending on the parameters.

Therefore, before optimization, a parameter (Exponent value k, population size, crossover probability P_c, mutation probability P_m) survey is carried out using the hole model. The values of $k=1$, population$=50$, $P_c=0.2$, $P_m=0.2$ are given as initial conditions. For the parameter survey, only one parameter was changed while the others were fixed. Then the best value for the changed parameter was selected.

The estimated best results for the hole model were already obtained by the object-oriented optimization method (maximum strain energy is reduced to 18%).

3. Setting Up Parameters

By using the hole model, optimization is carried out under the enforced displacement condition and various parameters are set up. The results are given below.

3.1 Selection probability (selection of k value)

If there are the individuals A and B, of which superiority values are 0.3 and 0.31, the ratio of selection probability is only 1.01 times with $k=1$, but 1.33 times and 1.53 times with $k=20$, 30. The longer the k value is, the faster the excellent individuals spread in the group. There is also the possibility, however, of falling into a local minimum. The results of prameter survey of k are shown in Fig. 7. The horizontal axis shows time and vertical axis shows the superiority value of the best chromosome. The following diagrammatic charts of parameter surveys have the same axes.

There is a tendency that the longer the k value is, the more rapidly the superiority value reduces. However, this is not always the case, because the result depends on a random number. Since the difference between $k=20$ and $k=30$ is small, $k=20$ is selected, because the superiority at this k value is the most rapidly reduced.

3.2 Population size

The analysis was conducted with population sizes of 5, 10, 50 and 100. The results of prameter servey of population size are shown in Fig. 8. There are about $4^{23} \fallingdotseq 10^{13}$ solutions and there is the possibility of falling into a local minimum if the population size is too small. If the population size is too big, however, it requires a long time for evaluation and the speed of evalution is slow. The result shows a tendency that the small population size is better than the larger one. The comparison was carried out with respect to time, because time is more important than generation number. Based on the above, the number of 10 is selected

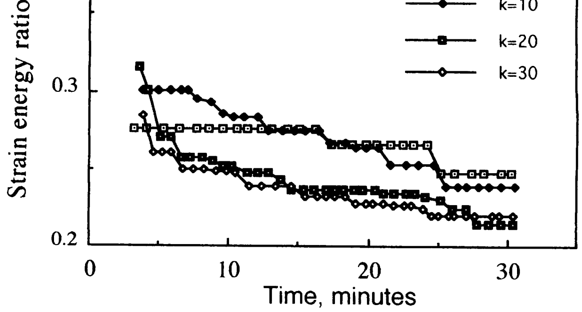

Fig. 7 Parameter survey of selection probability

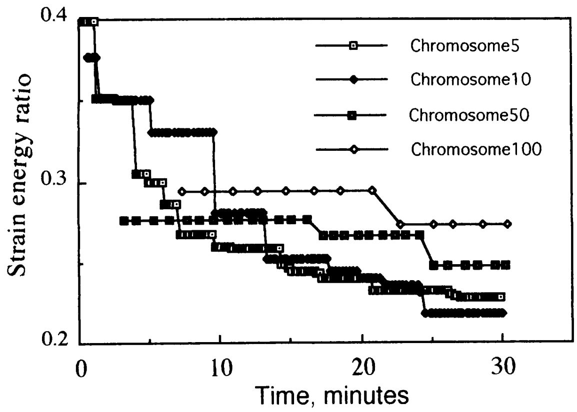

Fig. 8 Parameter survey of population size

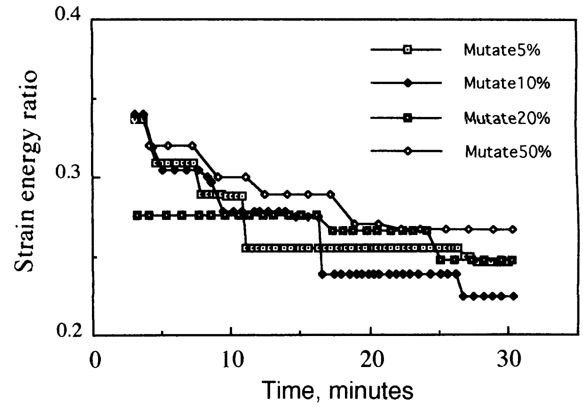

Fig. 10 Parameter survey of mutation probability

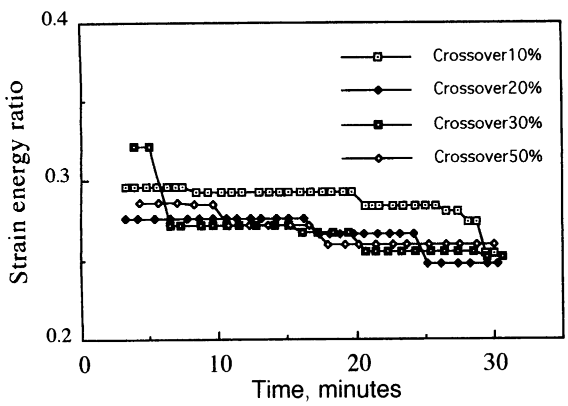

Fig. 9 Parameter survey of crossover probability

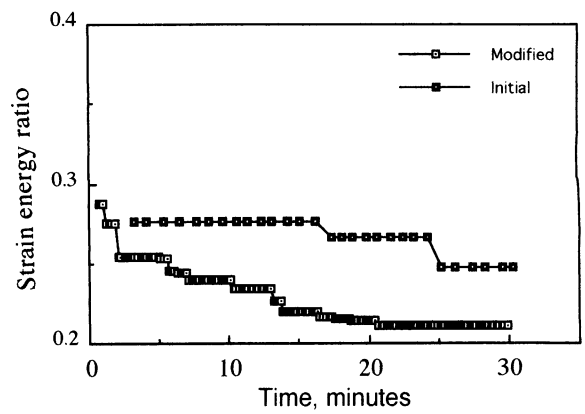

Fig. 11 The result after conducting parameter survey

as the population size in this study.

3.3 Crossover

The results for crossover probability P_c=10, 20, 30 and 50% are shown in Fig. 8. There is no large change by P_c=10%. There is no large difference in another P_c values. Based on the above, 20% is selected as the crossover probability in this study.

3.4 Mutation

The results for mutation probability P_m=5, 10, 20 and 50% are shown in Fig. 9. Although mutation depends on a random number and it is difficult to compare with each other results, it seems that searching is conducted only in a narrow region of all solutions if the P_c value is too small. If the P_c value is too large, conversely, the change of genes is too great and the calculation is not effective. Therefore, 10% is used for mutation probability P_c based on the results in Fig. 9.

3.5 Results of parameter survey and conclusion

The result after optimization of parameters is compared with those before it in Fig. 10. As shown in Fig. 10, a good result was obtained by means of the parameter survey.

Since genetic algorithms depend on a random number, a statistical method is necessary to evaluate their accuracy, but it is possible to obtain better results using the simple parameter survey (Fig. 11).

The conclusions are summarized as follows.

(1) It is better to select a superior individual based on high value of exponent k.

(2) It is better to use a small population and to increase number of generations.

(3) It is better to use relatively high probability of mutation.

(4) Crossover is not very important.

As an extreme example of the genetic algorithms obtained from these premater servey results, a random search method is examined. The parameters of the random search methods are the population size=1, the mutation probability P_m=100% and the crossover probability P_c=0%. Because of the elite preservation strategy, mutation is carried out on the same individual until a better individual is found. Due to the small population, the efficiency is very good, but the results depend on the initial individual. Moreover, the random search method has the possibility to fall easily into a local minimum. In order to investigate the effect of gene number on mutation, 2, 10 and 20% of genes are changed by mutation using the hole model. To maintain variety, grouping is not conducted in this

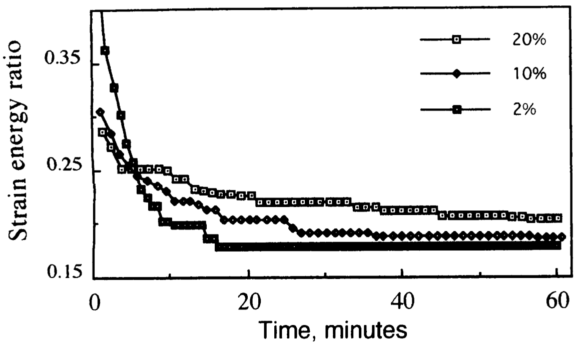

Fig. 12 Parameter survey of the number of mutated genes

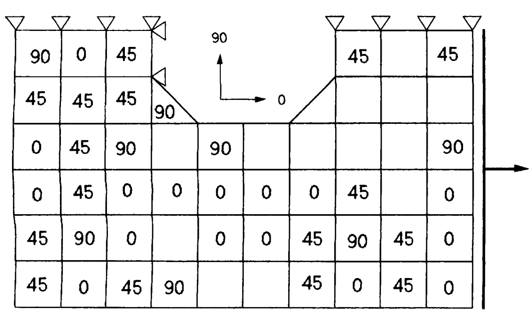

Fig. 13 The result of genetic algorithm

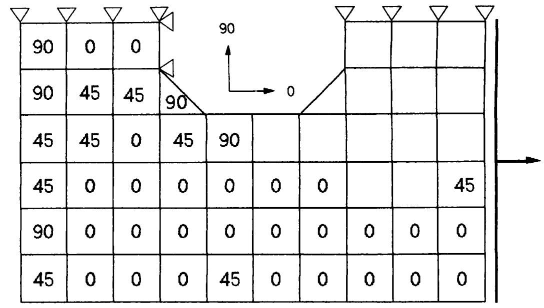

Fig. 14 The result of random search

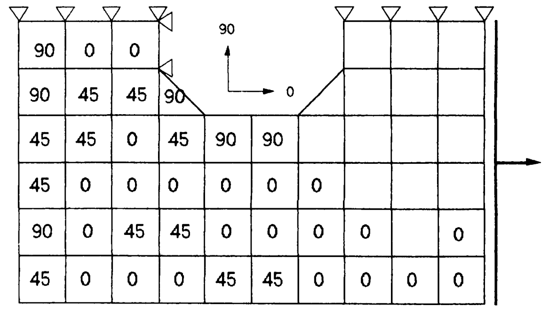

Fig. 15 The result of object-oriented optimization method

model. Therefore, the numbers of mutated genes are directly 2, 5 and 9, respectively. The results are shown in Fig. 12.

The result for 2% shows the best reduction of strain energy ratio to 18% in 17 min (Fig. 12). It is effective to change genes little by little, and it is shown that the method is better than genetic algorithms for the hole model.

4. Optimization of Bolt Model

The bolt model, shown in Fig. 2, was optimized by three methods: the genetic algorithms, the random search and the object-oriented optimization method developed by us[2]. The parameters of the genetic algorithms and random search were examined in section 3 above.

The results are shown in Figs. 13, 14 and 15, respectively. The numbers in figures as the angles of stiffener. The calculations of genetic algorithms and random search end in 12 h. As shown in Fig. 13, the angle distribution of stiffener determined by the genetic algorithms is not clear. The maximum strain energy was decreased to 33%. Conversely, the result of the random search shows a clearly divided distribution between the regions with and without stiffener. The maximum strain energy is reduced to 29%. The object-oriented optimization method requires 2 h 45 min and the maximum strain energy was decreased to 31%. From these results it can be concluded that random search is the best method.

Otherwise, by applying experience, isolated stiffeners shown in Fig. 13 are changed and removed, but maximum strain energy increased to about 31%.

Random search is proved to be an effective method, as mentioned above, for the optimization problem to reduce the strain energy concentration of a laminated composite plate by attaching stiffener, but it depends on the initial value and there was a case in which the maximum strain energy fell only to 31% after 54 h of optimization. Therefore simultaneous analysis using multiple systems is necessary.

In this problem, even if part of an excellent chromosome is changed by crossover between two individuals, the new individuals are not always excellent because the connection to another part of genes is also important. In other words, the angle distribution of stiffener of the whole structure is important and local optimization is meaningless. Therefore, for such problem, a random search, which is an extreme genetic algorithm, is better than the general genetic algorithms.

5. Conclusion

Genetic algorithms were applied to the optimization problem in which stiffener was attached to laminated composite plates to reduce strain energy

concentration of a hole model and a bolt model.
The summary of results is as follows.

(1) Optimization of the distribution of the stiffener was conducted using genetic algorithms.

(2) Parameters of genetic algorithms were selected by a simple parameter survey and the suitability was proved.

(3) A random search method, which is an extreme genetic algorithm, was proposed. This approach is effective for the problem when the distribution of genes is important and crossover is not efficient. The effectiveness of this approach for the hole model and the bolt model of this study was proved.

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