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Composite Structures 81 (2007) 419–426
COMPOSITE STRUCTURES
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New iteration fractal branch and bound method for stacking sequence optimizations of multiple laminates

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Available online 16 October 2006

Abstract

For laminated composite materials, the stacking sequence design is indispensable to make efficient use of the material properties. Laminated composites are usually fabricated from unidirectional plies of given thickness with limited fiber orientations to a small set of angles (e.g. 0°, +45°, 45°, and 90°). An improved version of fractal branch and bound method has been proposed to optimize multiple stacking sequences. The improved method, however, cannot be applied to complicated structures, since it is difficult to determine which is the most effective surface for designing such structures. In the present study, therefore, a new iteration fractal branch and bound method is proposed, which does not require empirical knowledge of the target structure. Here, this method is applied to a hat-stiffened panel to enable the stacking sequences of the two panel laminates and hat-type stiffener to be optimized simultaneously.

Keywords: Laminated Composites; Optimization; Stacking Sequence; Stiffened Panel; Buckling

1. Introduction

For laminated composite materials, the design of the stacking sequence is indispensable if an efficient use of the material properties is to be realized. Laminated composites are usually fabricated from unidirectional plies of given thickness, with fiber orientations limited to a small set of angles (e.g. 0°, +45°, 45°, and 90°). The problem of designing such laminates for various strength and stiffness requirements is an integer-programming problem of selecting a required number of plies of each orientation and then determining an optimal stacking sequence. Although the branch and bound method has occasionally been used to solve stacking sequence optimization problems[1], generally, it is genetic algorithms that are more popular for such problems [2–12].

Narita has published a new technology to design the optimal stacking sequence based on a layer-wise optimization method [13]. The layer-wise method optimizes a stacking sequence based on the properties of the outer plies, which have a greater effect on the overall stiffness of the laminates. We have already proposed a fractal branch and bound method [14–17] for optimizing stacking sequences of laminates. Essentially, this method adopts a response surface approximation by means of a quadratic polynomial on the basis of the lamination parameters. Unlike the conventional branch and bound method, the fractal branch and bound method is particularly economical with regards to computational cost due to the use of an approximation, making it highly efficient for pruning selection branches. The improved fractal branch and bound method was applied not only to buckling load maximizations, but also to flutter speed maximizations with multiple local maximums. The fractal branch and bound method, however, was deemed to be limited to addressing the stacking sequence optimizations of single laminates.

An improved version of the fractal branch and bound method is proposed to optimize multiple stacking sequences, and the method is applied to a hat-stiffened panel [18]. Here, the stacking sequences of the two panel laminates and hat-type stiffener of the hat-stiffened panel

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doi:10.1016/j.compstruct.2006.09.001

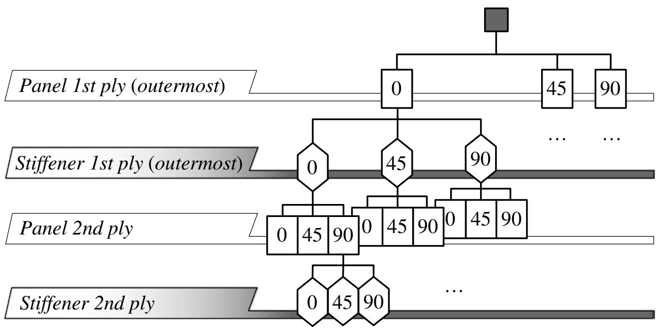

Fig. 1. Altenating stacking sequence optimization process of stiffened panel.

are optimized simultaneously, as shown in Fig. 1. For the optimization procedure, the outer laminate plies for the panel and hat-type stiffener are searched for alternately. While the improved method was successfully applied to the stiffened panel, it cannot be applied to more complicated structures or unsymmetrical laminates, because of the difficulty in determining the most effective outermost ply for these particular structures. For example, a hat-type stiffener has an additional laminate on top of the hat stiffener to prevent local buckling; the outermost ply of the hat stiffener is not the same as the outermost ply on top of the hat stiffener. This means that the orders of optimization for the separate components (i.e. the panels for the hat-type stiffener and the stiffener laminate) may have a large effect on the overall stacking sequence optimization.

In the present study, therefore, a new iteration fractal branch and bound method is proposed. This new method is very simple for software coding and can easily be applied to complicated structures. Here, the new method is applied to optimize the stacking sequences of a hat-type stiffened panel.

2. Schema of fractal branch and bound (FBB) method

The in-plane and out-of-plane stiffness moduli of symmetric laminates are expressed by means of lamination parameters. A symmetric laminate has four in-plane parameters (V_1^*, V_2^*, V_3^* and V_4^*) and four out-of-plane (W_1^*, W_2^*, W_3^* and W_4^*) lamination parameters. In the present study, we assume that the available fiber angles are limited to a small set (0°, 90° and ±45°). As such, the V_3^*, V_4^* and W_4^* parameters are effectively canceled and W_3^* tends to a small value due to the balance rule, leaving the remaining four-lamination parameters (V_1^*, V_2^*, W_1^*, W_2^*) for each laminate to be considered.

The in-plane and out-of-plane lamination parameters of the symmetric laminates of $2N$ plies (total number of plies is $2N$) are defined as follows:

$$\mathbf{V} = \begin{bmatrix} V_1^* \\ V_2^* \end{bmatrix} = \sum_{k=1}^{N} (a_{k-1}^V - a_k^V) \begin{bmatrix} \cos 2\theta_k \\ \cos 4\theta_k \end{bmatrix}, \quad a_k^V = \frac{N-k}{N}, \tag{1}$$

$$\mathbf{W} = \begin{bmatrix} W_1^* \\ W_2^* \end{bmatrix} = \sum_{k=1}^{N} (a_{k-1}^W - a_k^W) \begin{bmatrix} \cos 2\theta_k \\ \cos 4\theta_k \end{bmatrix}, \quad a_k^W = \left(\frac{N-k}{N}\right)^3, \tag{2}$$

where θ_k is the fiber angle of the kth ply from the outermost ply ($k = 1, \ldots, N$).

For optimization of the stacking sequence in the present study, eight lamination parameters for the skin panel and the stiffener are considered. The lamination parameters of the panel are expressed as $V_{1\mathrm{p}}^*$, $V_{2\mathrm{p}}^*$, $W_{1\mathrm{p}}^*$, and $W_{2\mathrm{p}}^*$, while the lamination parameters of the stiffener are expressed as $V_{1\mathrm{s}}^*$, $V_{2\mathrm{s}}^*$, $W_{1\mathrm{s}}^*$, and $W_{2\mathrm{s}}^*$.

The values of the trigonometric functions used in the lamination parameters in Eqs. (1) and (2) are limited to the cases expressed as follows due to the constraints:

$$\begin{bmatrix} \cos 2\theta_k \\ \cos 4\theta_k \end{bmatrix} = \overset{0^\circ}{\begin{bmatrix} 1 \\ 1 \end{bmatrix}} \quad \overset{\pm 45^\circ}{\begin{bmatrix} 0 \\ -1 \end{bmatrix}} \quad \overset{90^\circ}{\begin{bmatrix} -1 \\ 1 \end{bmatrix}}. \tag{3}$$

We can consider the lamination parameters as the elements of the in-plane and out-of-plane lamination parameter vectors **V** and **W**. The coefficients of the vectors in Eqs. (1) and (2) are all positive in value, and the total sum of the coefficients satisfies the equations as follows:

$$\sum_{k=1}^{N} (a_{k-1}^V - a_k^V) = 1, \quad \sum_{k=1}^{N} (a_{k-1}^W - a_k^W) = 1. \tag{4}$$

These equations signify that the lamination parameter values are limited to the boundary and internal area of a triangle, in which the tree vectors in Eq. (3) represent the tree apices.

Next, we consider the case where the outer d plies are chosen and the inner plies are unknown, where this laminate can be expressed as $[\theta_1/\theta_2/\cdots/\theta_d/*/*/\cdots/*]_\mathrm{S}$. Let us consider the area in the lamination parameter space in which these laminates exist. The lamination parameters can be expressed as follows:

$$\mathbf{V} = \mathbf{V}_0 + a_d^V \mathbf{V}', \quad \mathbf{W} = \mathbf{W}_0 + a_d^W \mathbf{W}', \tag{5}$$

$$\mathbf{V}_0 = \sum_{k=1}^{d} (a_{k-1}^V - a_k^V) \begin{bmatrix} \cos 2\theta_k \\ \cos 4\theta_k \end{bmatrix},$$
$$\mathbf{W}_0 = \sum_{k=1}^{d} (a_{k-1}^W - a_k^W) \begin{bmatrix} \cos 2\theta_k \\ \cos 4\theta_k \end{bmatrix}, \tag{6}$$

$$\mathbf{V}' = \frac{1}{a_d^V} \sum_{k=d+1}^{N} (a_{k-1}^V - a_k^V) \begin{bmatrix} \cos 2\theta_k \\ \cos 4\theta_k \end{bmatrix},$$
$$\mathbf{W}' = \frac{1}{a_d^W} \sum_{k=d+1}^{N} (a_{k-1}^W - a_k^W) \begin{bmatrix} \cos 2\theta_k \\ \cos 4\theta_k \end{bmatrix}. \tag{7}$$

Since $\mathbf{V}_0$ and $\mathbf{W}_0$ in Eq. (6) depend only on the determined fiber angles, these values can be obtained exclusively. The fiber angles of $\boldsymbol{V}'$ and $\boldsymbol{W}'$ in Eq. (7) are not chosen. In Eq. (7), all of the coefficients are positive in value and the total sum of the coefficients is equal to unity. This means that the feasible values of $\boldsymbol{V}'$ and $\boldsymbol{W}'$ in Eq. (7) are limited to the internal area and edges of a triangle constructed from the three vectors defined in Eq. (3). By definition, the coefficients of a_d^V and a_d^W are positive in value and all smaller than unity. This implies that the feasible laminates are located at the edges and internal area of a second triangle whose center is located in the lamination parameter space at $(\mathbf{V}_0, \mathbf{W}_0)$. Moreover, the lengths of the triangle sides are shorter by a factor of a_d^V and a_d^W, respectively.

As the number of fixed plies d increases, the coefficients a_d^V and a_d^W decrease. The decrease in a_d^V and a_d^W is indicative of shrinkage in the self-similar triangle. This process results in a fractal pattern of the feasible laminates in the lamination parameter space (Fig. 2), produced by plotting all of the 12-ply symmetric laminates individually. In Fig. 2, the small triangle set inside the larger triangle represents the domain of the laminate set represented by means of the stacking sequence of $[0/45/90/*/*/*]_S$.

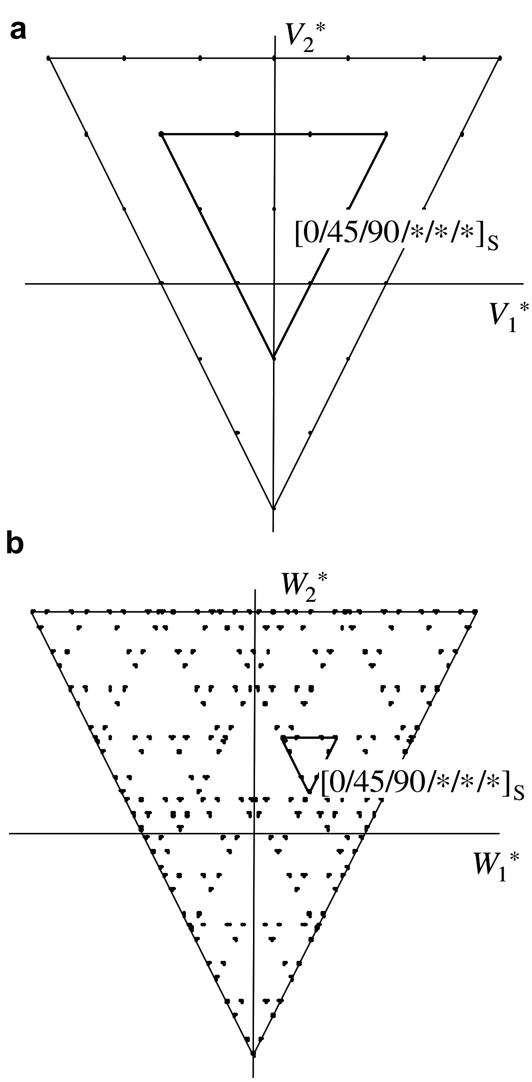

Fig. 2. Fractal pattern drawn by plotting all feasible laminates of 12 plies ($N = 6$): (a) in-plane lamination parameter coordinates, (b) out-of-plane lamination parameter coordinates.

The fractal branch and bound method was proposed as a fast and deterministic optimal stacking sequence search method [14–18], which requires an approximation of the design space, realized using a quadratic polynomial of the lamination parameters as explanatory variables for the response surface. This method is limited to providing deterministic optimal stacking sequences for just single laminates over short periods of time.

Let us consider the case where the response surface of the approximated objective function is obtained for a single laminate. In this formula, **V** and **W** are the vectors comprising the in-plane and out-of-plane lamination parameters, respectively:

$$f(\mathbf{V},\mathbf{W}) = c + [\mathbf{V}^{\mathrm{T}}\mathbf{W}^{\mathrm{T}}]\begin{bmatrix} \boldsymbol{b}_V \\ \boldsymbol{b}_W \end{bmatrix} + \frac{1}{2}[\mathbf{V}^{\mathrm{T}}\mathbf{W}^{\mathrm{T}}] \times \begin{bmatrix} \boldsymbol{A}_{VV} & \boldsymbol{A}_{VW} \\ \boldsymbol{A}_{VW}^{\mathrm{T}} & \boldsymbol{A}_{WW} \end{bmatrix}\begin{bmatrix} \mathbf{V} \\ \mathbf{W} \end{bmatrix}. \tag{8}$$

Let us consider a general case of $[\theta_1/\theta_2/\theta_3/\cdots/\theta_d/*/*/\cdots/*]_S$, where the outer d plies are predetermined. This case creates a triangle expressed as Eqs. (5)–(7). The objective function can therefore be expressed as follows:

$$\begin{aligned}
f &= f(\mathbf{V},\mathbf{W}) = f(\mathbf{V}_0 + a_d^V\mathbf{V}', \mathbf{W}_0 + a_d^W\mathbf{W}') \\
&= f_0 + f_V' + f_W' + f_{VW}', \quad f_0 = f(\mathbf{V}_0, \mathbf{W}_0), \\
f_V' &= a_d^V \mathbf{V}'^T \boldsymbol{b}_V' + (a_d^V)^2 \frac{1}{2}\mathbf{V}'^T[A_{VV}]\mathbf{V}', \\
f_W' &= a_d^W \mathbf{W}'^T \boldsymbol{b}_W' + (a_d^W)^2 \frac{1}{2}\mathbf{W}'^T[A_{WW}]\mathbf{W}', \\
f_{VW}' &= a_d^V a_d^W \mathbf{V}'^T[A_{VW}]\mathbf{W}', \\
\begin{bmatrix} \mathbf{b}_V' \\ \mathbf{b}_W' \end{bmatrix} &= \begin{bmatrix} \mathbf{b}_V \\ \mathbf{b}_W \end{bmatrix} + \begin{bmatrix} A_{VV} & A_{VW} \\ A_{VW}^{\mathrm{T}} & A_{WW} \end{bmatrix}\begin{bmatrix} \mathbf{V}_0 \\ \mathbf{W}_0 \end{bmatrix}.
\end{aligned} \tag{9}$$

Eq. (9) means that the response f can be represented by the sum of the four terms as follows:

f_0: constant
f_V'': quadratic polynomial of $\mathbf{V}'$
f_W'': quadratic polynomial of $\mathbf{W}'$
f_{VW}'': linear interaction term of $\mathbf{V}'$ and $\mathbf{W}'$

The conservative evaluation **g** of the objective function f is defined as follows:

$$\mathbf{g} = f_0 + \max(f_V') + \max(f_W') + \max(f_{VW}'). \tag{10}$$

In order to obtain an evaluation on the outside value of **g**, **V** and **W** are assumed to be independent of each other. This maximization is performed within each of the reduced triangular regions, where the maximum values of f_V'' and f_W'' are easily calculated as these are a simple quadratic polynomial bound by the triangle. Since f_{VW}'' is a linear interaction term, the maximum value of f_{VW}'' is located at the apexes of the triangle.

Therefore, the value of Eq. (10) can be easily calculated. Since **V** and **W** are not independent variables, the actual

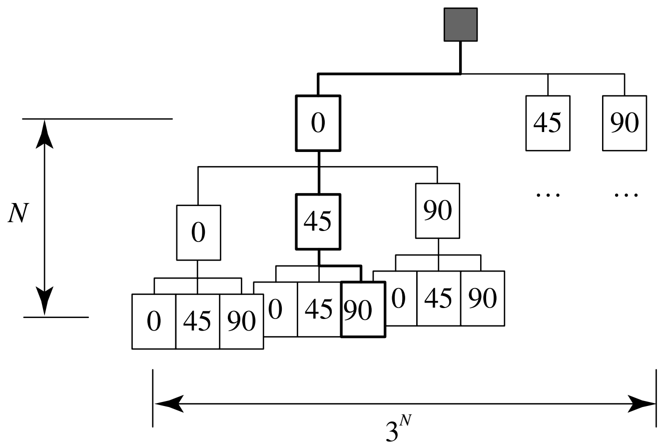

Fig. 3. Tree structure of a stacking sequence.

maximum value of the response surface is surely smaller than the value of **g**. This means that Eq. (10) can be used as an evaluation function for the branch and bound method.

When each laminate is evaluated, the infeasible laminates violating the stacking sequence constraints, such as the four-contiguous ply rule, can be eliminated. The stacking sequence constraints, therefore, can be easily implemented here.

Evaluations of the maximum **g** value from Eq. (10) require only maximization of the quadratic polynomials in the reduced triangular region, obtained from the fractal arrangement analysis of feasible laminates. This means that the estimation can be applied for non-convex problems. When the response surface falls within a certain degree of accuracy, the described optimization method provides global optimal laminate.

Since the branches of feasible laminates create fractal patterns, as previously described, the pruning of these fractal pattern branches to obtain an optimal stacking sequence is named a fractal branch and bound method.

The optimal stacking sequence is a feasible laminate that gives the maximum response *f*. The stacking sequence of each laminate can be represented as a tree-structure chart (as shown in Fig. 3) in search of the fiber angles of the outer plies. Since there is no difference in the lamination parameters if the 45°-plies are substituted for −45°-plies, all of the angle plies in the tree structure are expressed as 45°-plies. The fractal branch and bound method is a simple method for ascertaining the optimal stacking sequence. For fast searching, an efficient pruning method for searching the tree branch is indispensable. Here, the fractal branch and bound method uses fractal patterns that are obtained when all of the feasible laminates are plotted in the lamination parameter space so that the number of search branches can be reduced. This enables optimization to be performed in a much shorter space of time. A more detailed description is given in Ref. [15]. The FBB method has already been extended to unsymmetrical laminates in our previous study [19].

3. New iteration FBB method for optimization of multiple laminates

Let us consider the case where the total number of laminates to be optimized is p. In this case, each laminate has two in-plane and two out-of-plane lamination parameters: vectors of $\mathbf{V}_i$ and $\mathbf{W}_i$ $(i = 1, \ldots, p)$ have elements of (V^*_{i1}, V^*_{i2}) and (W^*_{i1}, W^*_{i2}). In order to use the fractal branch and bound method, we have to obtain an approximated objective function by means of a quadratic polynomial as follows:

$$f = f(\mathbf{V}_1, \mathbf{W}_1, \ldots, \mathbf{V}_p, \mathbf{W}_p). \tag{11}$$

The coefficients of the quadratic polynomial in Eq. (11) are obtained using the well-known least square errors method. For the calculation, the design of experiments in the lamination parameter space. In the present study, D-optimal laminates are selected for calculating the line of regression for the least square errors method. The design of experiments of the lamination parameter space is described in the references [20]. Once Eq. (11) has been established, the optimization process can be initiated.

In a previous study, searching of the optimal stacking sequence was initiated from the outermost ply of the panel, as shown in Fig. 1, and once obtained, the search was then directed toward establishing the outermost ply of the stiffener. This alternating process of optimizations continues until all of the plies are optimized.

In the new iteration FBB method, provisional stacking sequences of the laminate number are determined randomly for 2 to p. When all the other laminates are determined, the last remaining stacking sequence in Eq. (11) corresponds to laminate (1), with coordinates $(\mathbf{V}_1, \mathbf{W}_1)$. For the stacking sequence optimization of a single laminate, it is possible to use the conventional FBB method. After obtaining the optimal laminate, both the optimal set of laminates and the objective function are compared with those of the provisional set. If the objective function and provisional set of laminates are different from those of the obtained optimal sets of laminates, then the obtained optimal set of laminates is saved in the new provisional sets of laminates. The stacking sequence of laminate (1) is fixed to the obtained optimal laminate, and the target is moved to the second laminate position. This process is cycled until the conversion corresponds to that shown in Fig. 4.

Iteration of the target laminate using this new method optimizes only a single laminate during each process. This process makes the software coding very simple, such that an empirical knowledge of the optimization process is not required: the new iteration FBB method is appropriate for the automatic optimization of even complicated structures. Of course, there is a possibility of obtaining a local optimal as the alternating FBB method adopts the iteration process. Since the FBB method can be completed within a very short space of time, it is possible to perform several optimizations without spending too much time changing the initial provisional set of the laminates.

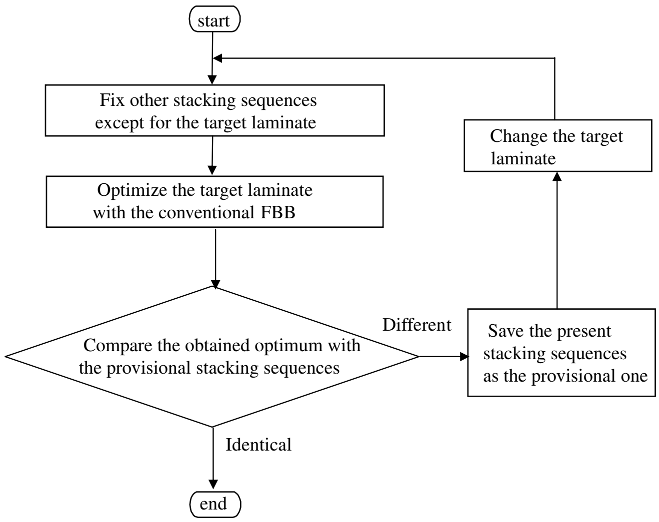

Fig. 4. Flow chart of the new alternating FBB method.

This alternating FBB method is capable of processing several optimal sets of laminates automatically, until the search matches an identical set of laminates to those already found in the list of optimal sets of laminates.

4. Application to a stiffened panel problem

4.1. Optimization problem

The new iteration FBB method is applied to an optimization of stacking sequences for a hat-stiffened panel (the configuration of which is shown in Fig. 5) constructed from T300/5208, which has the material properties given in Table 1. The dimensions of the stiffened panel are obtained from the optimization results given in Ref. [21], and listed in Table 2. The boundary conditions of the hat-stiffened composite structure are determined similarly to the analysis performed by Bushnell and Bushnell [22]. In Fig. 4, the parameters u_x, u_y and u_z correspond to the displacement in each axis direction at the boundaries, while rot_x, rot_y and rot_z correspond to the rotation around the axes at these boundaries. The reference compression load λNx is applied to line $x = 0$. In the present study, the finite element analyses are performed using the commercially available FEM code ANSYS (ver.8). These particular laminated structures are modeled using the ANSYS SHELL99 element, where the total number of grids and elements used are 8858 and 2800, respectively. The model laminate structures are meshed using the ANSYS auto mesh method

4.2. Response surface approximation

For the stiffened panel, the target laminates correspond to the panel and the hat-stiffener. These two laminates have a total of eight lamination parameters as variables: $V^*_{1p}, V^*_{2p}, W^*_{1p}$ and W^*_{2p} (panel laminate); $V^*_{1s}, V^*_{2s}, W^*_{1s}$ and W^*_{2s} (stiffener laminate). When a full quadratic polynomial is used for the eight variables, the total number of unknown coefficients is 45 $(=1 + 8 + 8 + 8 \times 7/2)$. From empirical knowledge, the total number of analyses required to calculate the unknown coefficients using the least square errors method should be almost double the total number of coefficients. In the present study, therefore, 90 analyses are selected by means of D-optimal laminates, the details of which are given in Ref. [20]. In the present study, however, to reduce the total number of candidate laminates, the half ply number used for making the candidate laminates is four.

Using ANSYS, the buckling load ratio λ is calculated at each selected 90 laminate, as well as at the laminate located at the origin of the lamination parameter space ($\mathbf{V} = \mathbf{W} = 0$). The origin of the lamination parameter space is added to reduce the bias of the response surface at its center. This laminate corresponds to an ideal, perfectly isotropic laminate.

Using the 91 FEM results, the response surface to approximate the buckling load ratio of a quadratic polynomial is calculated. To reduce the bias around the maximum buckling load and the center of the lamination parameter space, the results of the two superior laminates and the origin are added three times for the least square errors method. Therefore, the total number of data points for the least square errors method is 97. The obtained quadratic polynomial is as follows:

$$\begin{aligned} y = {} & 4.451 - 1.987x_1 - 1.060x_2 - 0.6633x_4 + 2.342x_5 \\ & + 0.4612x_6 - 0.2378x_8 + 2.810x_1^2 + 0.5904x_1x_2 \\ & - 2.314x_1x_3 - 1.163x_1x_5 + 0.3706x_1x_7 \\ & + 0.4702x_1x_8 - 0.4692x_2x_3 + 0.5032x_2x_4 \\ & + 0.4337x_3x_4 - 0.4725x_4^2 + 0.3534x_4x_8 - 1.434_5x_6 \\ & - 1.196x_5x_7 + 0.5097x_6x_7 - 0.9049x_6x_8 \\ & - 0.7577x_7x_8 + 0.4271x_8^2, \end{aligned} \tag{12}$$

where $x_1 = V^*_{1p},\ x_2 = V^*_{2p},\ x_3 = W^*_{1p},\ x_4 = W^*_{2p},\ x_5 = V^*_{1s},\ x_6 = V^*_{2s},\ x_7 = W^*_{1s},\ x_8 = W^*_{2s}$.

Here, the adjusted determination coefficient is$R^2_{adj} = 0.903$. This means the response surface well approximates the buckling load. The estimation is shown in Fig. 6, where the abscissa corresponds to the calculated buckling load, determined by means of FEM analyses, and the ordinate represents the estimated buckling load by means of a quadratic polynomial. The accuracy of the estimation is confirmed by the minimal deviation of the plotted (XXX) from the diagonal line. The triangular symbols correspond to the two superior laminates used three times for regression purposes to reduce the bias around the maximum buckling load laminate. From this figure, we can recognize that the bias around the maximum buckling load laminate is relatively small compared to those of other laminates in the area around the high buckling load.

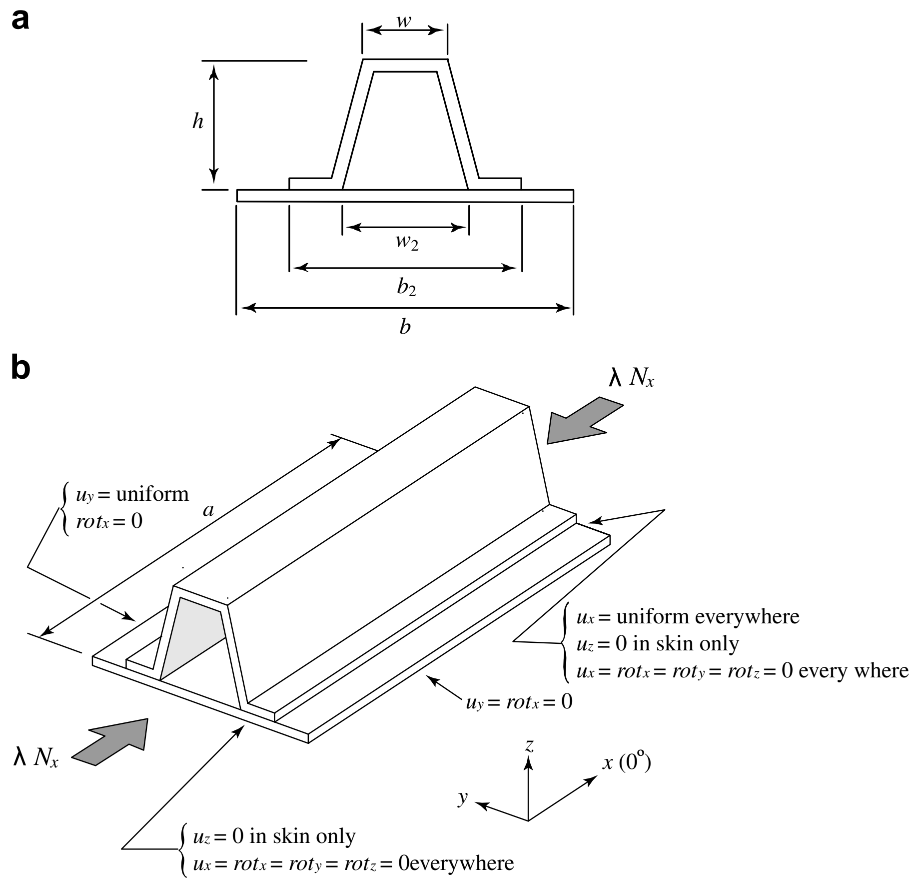

Fig. 5. Analysis model of hat-stiffened panel. (a) Cross section geometry of hat-stiffened panel. (b) The whole view of analysis model.

Table 1
Material properties used in the present study (T300/5208)

E_L	181 GPa
E_T	10.3 GPa
G_{LT}	7.17 GPa
G_{TZ}	3.72 GPa
ν_{LT}	0.28
ν_{TZ}	0.386

Table 2
Dimensions of the hat-stiffened panel

a	1 m
b	0.25 m
c	0.16 m
w	0.05 m
w_2	0.11 m
h	0.05 m

4.3. Results and discussion

Using the proposed new iteration method and the obtained response surface, the stacking sequences of two laminates are optimized. Each of the panels and stiffeners are made from 16-ply laminates: the half number of plies is therefore 8 for both laminates. For the optimization step, two rules are adopted: the balance rule and the four-contiguous-ply rule, as used in Ref. [7]. The obtained optimal stacking sequences are listed in Table 3. In this table, the buckling load ratios obtained from the response surface and the FEM analysis are shown, where the values obtained from the response surface are just 2.3% smaller than those produced using the FEM analysis. This implies that the response surface is completely reliable.

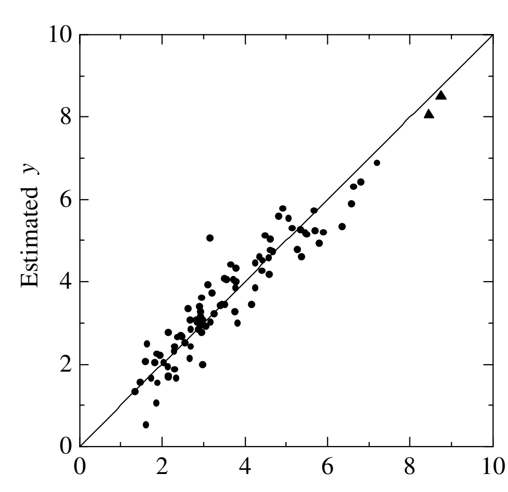

Fig. 6. Estimated response of the buckling load.

Table 3
Optimal stacking sequences using a global response surface

Stacking sequences		RS	FEM
Panel skin	$[\pm45/45/90_2/-45/90_2]_S$	8.123	8.312
Stiffener	$[\pm45/45/0_2/-45/0_2]_S$		

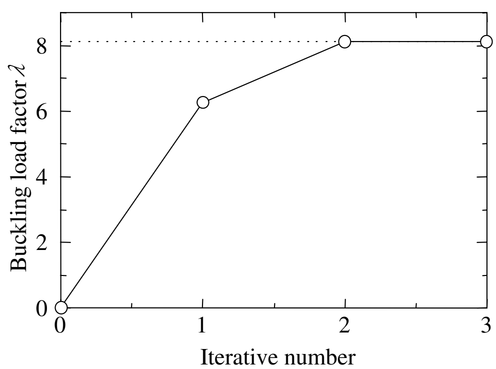

Fig. 7. Buckling load factor λ versus iterative number.

Table 4
Optimal stacking sequences using a modified response surface

Stacking sequences		FEM
Panel skin	$[(\pm45)_4]_S$	8.816
Stiffener	$[(\pm45)_2/0/90/0_2]_S$	

Fig. 7 shows the results of the iteration method, where the ordinate is the buckling load ratio estimated from the response surface, and the abscissa is the iterative number. Fig. 7 shows that optimal stacking sequences can be obtained from only four iterations.

Since the FBB method provides true optimal stacking sequences, the optimality error is dependent on the response surface errors in this method. To reduce the errors of the response surface near the optimal laminate, it is necessary to redesign the response surface using more FEM analyses near the optimal laminate. Therefore, the modified response surface method proposed in Ref. [23] is adopted here. 80 FEM analyses were performed in order to improve the estimation error near the optimal laminate. The corresponding results obtained from the modified response surface (Table 4) show a marked improvement in the buckling load (5.7%) compared with the values listed in Table 3. This means the first result is practical for optimal stacking sequences within an error of approximately 5%.

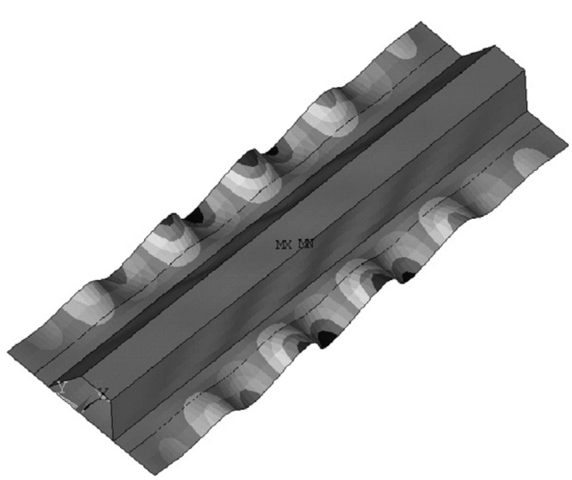

Fig. 8. Buckling mode shape of quasi-isotropic laminates. Skin: $[45/0/90/-45/90/-45/45/0]_S$, stiffener: $[45/0/90/-45/90/-45/45/0]_S$, buckling load ratio: $\lambda = 4.253$ (FEM).

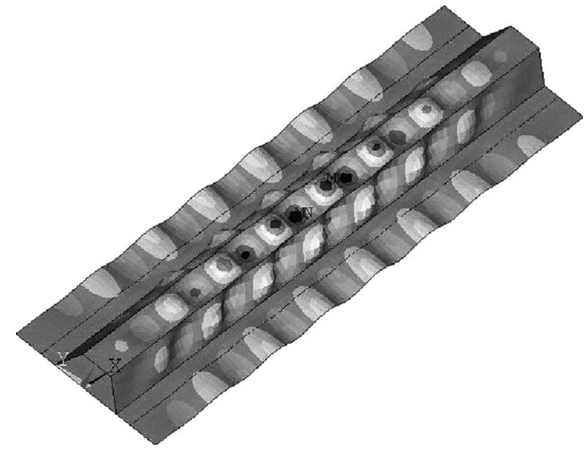

Fig. 9. Buckling mode shape of the true optimal laminates. Skin: $[(45/-45)_4]_S$, stiffener: $[(45/-45)_2/0/90]_S$, buckling load ratio: $\lambda = 8.816$ (FEM).

Fig. 8 shows the result of buckling load on quasi-isotropic laminates, while Fig. 9 reveals the true optimal laminates obtained from Table 3. The buckling load ratio for true optimal laminates is improved by 90% compared with the ratio obtained for quasi-isotropic laminates. The buckling mode itself is also improved: the entire structure deforms for optimal laminates.

5. Concluding remarks

A new iteration FBB method is proposed in the present study, and is applied to the stacking sequence optimizations of a hat-stiffened panel with two laminates. To obtain practical optimal stacking sequences, the new method required only 92 FEM analyses (including the recalculation of the optimal laminates). Using an additional 80 FEM analyses, it is shown that the true optimal stacking sequences can be obtained by means of the new iteration method. Unlike the alternating method, the new iteration method can be applied to the optimization of multiple stacking sequences for complicated laminated structures.

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