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Modified Efficient Global Optimization for a Hat-Stiffened Composite Panel with Buckling Constraint

Akira Todoroki* and Masato Sekishiro†
Tokyo Institute of Technology, Tokyo 152-8552, Japan

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The optimization method for composite structural components described herein uses modified efficient global optimization with a multi-objective genetic algorithm and a kriging response surface. For efficient global optimization using kriging, the kriging response surface is used as a representative of the function value. The stochastic distribution of the kriging is used to improve the estimation error of the kriging surrogate model. Using efficient global optimization, a hat-stiffened composite panel was optimized to reduce the weight with the buckling load constraint. The expected improvement was used as a single objective function of a particle swarm optimization. Nevertheless, it is difficult to obtain a feasible solution that satisfies buckling load constraints with the progress of optimization. Using a multi-objective genetic algorithm, we obtain the feasible optimal structure satisfying the constraints. The expected improvement objective function is divided into two objective functions: weight reduction and the uncertainty of satisfaction of the buckling load constraint. Kriging approximation, which is improved with the selected Pareto optimal frontier, reduces the computational cost. Also, a genetic algorithm is used to optimize the stiffened panel configuration. The fractal branch-and-bound method is used for stacking sequence optimizations. This method obtained a feasible optimal structure at a low computational cost.

Nomenclature

a = length of stiffened panel structure
b = spacing between stiffeners
b_2 = width of stiffener
h = height of stiffener
N_p = number of panel plies
N_s = number of stiffener plies
t_p = thickness of panel
t_s = thickness of stiffener
W = weight of structure
w = width of crown top
w_2 = width of crown bottom
$\mathbf{x}$ = variable vector
$\boldsymbol{\eta}$ = parameter vector of the kriging response model
n_s = number of finite element method computations to make the kriging response surface
μ = averaged value
ρ = density of composites
σ = standard deviation

I. Introduction

AEROSPACE structural components made from laminated composites require stacking sequence designs as well as dimensional designs because the mechanical properties of the components are highly dependent on the laminates' stacking sequences. Aircraft components are designed using the building block approach described in MIL-Handbook-17. That design approach necessitates numerous experiments while limiting the available fiber orientations to 0, 45, −45, and 90 deg to prevent high design costs. Consequently, the stacking sequence optimization becomes a combinatorial optimization with limited fiber angles. It is necessary to optimize stacking sequences of a laminated composite structure made by stacking unidirectional plies. For the stacking sequence optimization problem, Miki [1] and Fukunaga and Chou [2] proposed a graphical optimization method using lamination parameters. Genetic algorithms are the most popular method for the optimizations [3–11]. In fact, Narita [12] proposed a layerwise-optimization method that requires the lowest computational cost.

The authors have published a new deterministic optimization method for the stacking sequence of the composite laminates, the fractal branch-and-bound (FBB) method, which is applied to buckling load maximization problems [13,14]. The FBB method is also applied to a flutter speed maximization problem of a laminated composite wing that has multiple local maxima. It is extended to the stacking sequence optimization of multiple laminates, such as stiffened panels [15,16] and nonsymmetric laminates [17].

The FBB method uses a quadratic polynomial to approximate the objective function with the lamination parameters as variables for the quadratic polynomial. The use of the quadratic polynomial rapidly prunes useless branches of a search tree of stacking sequences. The FBB method, however, is inapplicable to optimization problems that deal with dimensions of the target structures, such as stiffened panels. Dimensions of the stiffener or the panel are the design variables for practical laminated composite structures as well as for the stacking sequences of the stiffeners and the panel. Therefore, the dimensions and laminates must be optimized together to obtain the true optimal structure. The optimization of the dimensions and laminates of the laminated composite structure is an extremely difficult problem because it must address both the continuous variables of the dimensions and the discrete variables of the stacking sequences, with simultaneous consideration of some constraints. Several papers have described the optimization of composite structures [18,19].

The method presented herein uses the efficient global optimization (EGO) with a kriging response surface (RS) to reduce the number of finite element method (FEM) computations [20]. Actually, EGO uses expected improvement (EI) to improve the surrogate model approximation; the EI is also used as an objective function. In our previous study, particle swarm optimization (PSO) was used as an optimizer, and EGO was used with PSO to optimize a hat-stiffened composite panel [21]. In that study, it was difficult to find optimal results that satisfied the constraint with the progress of the optimization process of PSO when the buckling constraint was

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*Professor, Department of Mechanical Sciences and Engineering, 2-12-1, O-okayama, Meguro; atodorok@ginza.mes.titech.ac.jp. Member AIAA.
†Graduate Student, Department of Mechanical Sciences and Engineering, 2-12-1, O-okayama, Meguro. Student Member AIAA.

severe for the given design space and when the surrogate model was not sufficiently exact. An improved modification is performed here to improve the efficiency of the optimization process of EGO. For this study, the value of the EI is separated into two objective functions (the weight reduction and the probability of the satisfaction of the constraint); the multi-objective genetic algorithm (MOGA) is used to solve this multi-objective optimization problem: the total structural weight of the target-stiffened panel is minimized with the constraint of the buckling load. In this study, kriging model approximation is used to reduce the computational cost of the genetic algorithm; the error of the kriging model is improved similarly to the process in EGO without using the value of the EI directly. The structural weight is the first objective function and the probability of satisfaction of the buckling load is the second objective function here. This method resembles the filter method proposed by Fletcher [22]. The second objective function in this study, however, is the probability of satisfaction of the constraint and the use of the Pareto frontier to select additional analyses to improve the kriging model. The structural weight is calculated easily in this study. Therefore, the weight reduction is calculated directly and the buckling load ratio is approximated using the kriging model. For the kriging model, the design and analysis of computer experiments (DACE) model [23] is used because the process of producing the kriging model is performed automatically with software coding.

From the kriging model of the buckling load ratio, the probability of satisfaction of the buckling load constraint can be calculated. The kriging model is an approximation model that includes estimation error. Therefore, this study specifically examines the probability of satisfaction of the constraint instead of the buckling load ratio obtained directly from the kriging model, as suggested in EGO [21].

This method provides candidates of a superior set of composite structures that have higher probability of satisfying the constraint. After computations of several top candidates selected from this set, exact buckling load ratios are obtained and the kriging model is improved. Using the improved kriging model, the optimal composite structures are obtainable at a lower computational cost.

II. Optimization Problem

The optimization target is the hat-stiffened composite panel presented in Fig. 1. The stiffener design variables are as follows: the stiffener height is h; the crown width is w; the hat's lower end width is w_2; the total width of the hat stiffener is b_2; and the number of stiffener plies is N_s. For the panel, the number of plies of the composite panel is the only design variable. The panel length is fixed to $a = 1$ m; the periodic structural spacing between the substructures of the hat-stiffened panel is fixed at $b = 0.25$.

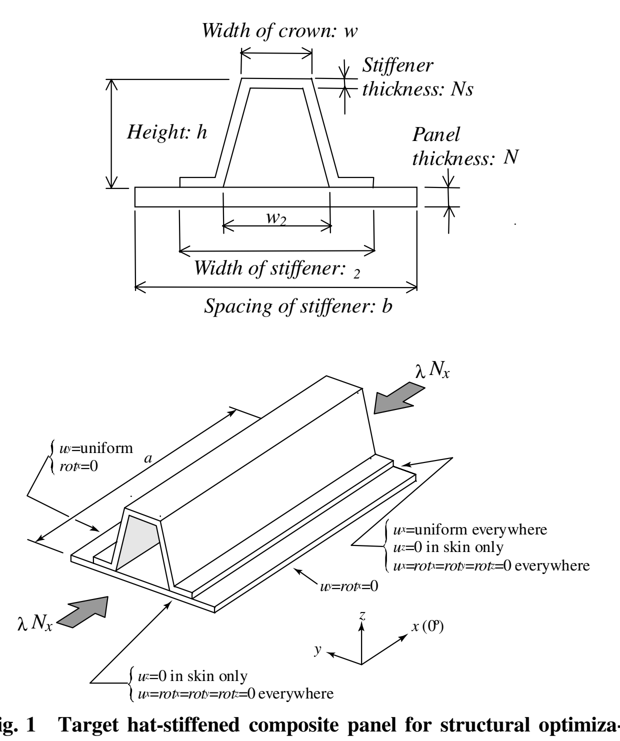

Fig. 1 Target hat-stiffened composite panel for structural optimization.

The buckling load is defined as a buckling load ratio (λ) against a single compression load to the x axis using the commercially available FEM code ANSYS; the load is $N_x = 750$ kN/m. For FEM analysis, a linear FEM analysis is used here with the eight-node linear shell element, SHELL99; the number of elements is 2800 and the total number of nodes is 8858. An automatic mesh system of ANSYS is used here. The boundary condition for the FEM analysis in the y direction is a repeating structure, as presented in Fig. 1; the y-axis edges are free, but rotation around the x axis is fixed to zero. A linear buckling analysis method is used here without using imperfection. Although the buckling load analysis does not exactly coincide with the experimental result, this analysis is simple and appropriate for the verification of the optimization process. The edges of the loading (x axis) have a fixed grip. This optimization problem is expressed in the formulas shown here.

$$\text{minimize } W \text{ subject to } \lambda \geq 1 \qquad b_2 - w_2 \geq 0.04 \tag{1}$$

Those expressions show the objective function W with structural weight; λ is the buckling load ratio against the reference load. The hat-stiffened panel is made from thin plates. Therefore, the total weight of the substructure is calculated simply from the dimensions of the hat-stiffened panel:

$$W = \{bt_p + (b_2 - w_2 + w + \sqrt{(w_2 - w)^2 + 4h^2})t_s\}\rho a \tag{2}$$

In Eq. (1), the first constraint means that the buckling load must be greater than 750 kN/m. The second constraint means that the width of the flange of the hat stiffener must be greater than 40 mm for processing. The available range of each design variable is presented in Table 1. The second constraint is satisfied automatically, meaning that the only constraint in the design is the buckling load ratio. The material used here is carbon-fiber-reinforced epoxy composite T300/5208. Its material properties are presented in Table 2. The composite panel and the hat stiffener are symmetric laminates. The available fiber angles are limited to four angles: 0, ±45, and 90 deg. For the fiber angles, there is a balance rule for the angle plies and a four-contiguous-ply rule to prevent large matrix cracks, that is, more than four plies must not be stacked when the fiber angle of each ply is the same.

III. Optimization Method Using the Kriging Model and the Multiple-Objective Genetic Algorithm

A. Approximation Using the Kriging Model

The true objective function of the optimization problem (1) is simple and easily calculated when all dimensions of the stiffened

Table 1 Design variables

Design variable	Min.	Max.
h	0.02	0.06
w	0.02	0.06
$w2$	0.08	0.12
N_p	4	16
N_s	4	16
$(b_2 - w_2)$	0.04	0.08

Table 2 Material properties of T300/5208

E_L	181 GPa
E_T	10.3 GPa
G_{LT}	7.17 GPa
ν_{LT}	0.28
ρ	1.6×10^3 kg/m^3

panel are given. The buckling load constraint, however, demands a large computational cost because it requires FEM analysis at each evaluation of the buckling load. In this study, the objective function of Eq. (1) is used directly without using the approximation model because it can be calculated easily. On the other hand, the constraint of the buckling load ratio is approximated using a kriging model to reduce the computational cost. As response surface variables, all dimensions of the hat-stiffened panel and the lamination parameters of both the panel and stiffener are used here. The automatic process to make a kriging model is very useful. Therefore, the DACE model is used.

A kriging model response surface based on the DACE model that comprises a response y and variable vector $\mathbf{x}_i$ $(i = 1, \ldots, k)$ can be expressed as

$$\hat{y}(\mathbf{x}) = \mu + Z(\mathbf{x}) \tag{3}$$

where all variables are normalized from -1 to $+1$ for n sets of analyses, n sets of responses, y_j $(j = 1, \ldots, n)$, and variables x_{ij} $(i = 1, \ldots, k$ and $j = 1, \ldots, n)$. In addition, μ is a constant value that is the averaged value of the global design space; $Z(\mathbf{x})$ is the variation from the averaged value at point vector $\mathbf{x}$.

Herein, $Z(\mathbf{x})$ is expressed as a realization of a stochastic process of the normal distribution of the mean value of zero. A response value y at point vector $\mathbf{x}$ is calculated as a correlation between the n sets of analysis results. The covariance matrix is given as

$$Cov[Z(\mathbf{x}^i), Z(\mathbf{x}^j)] = \sigma^2 \mathbf{R}[R(\mathbf{x}^i, \mathbf{x}^j)] \tag{4}$$

where $\mathbf{R}$ is the correlation matrix; element $R(\mathbf{x}_i, \mathbf{x}_j)$ is a value of a Gaussian correlation function between the points vectors $\mathbf{x}_i$ and $\mathbf{x}_j$. In this study, a distance between two points of each variable might not be expressed on a similar scale, which suggests the use of a scaled distance using multiplexing a constant to a distance, and the Gaussian correlation function can be written as

$$R(\mathbf{x}^i, \mathbf{x}^j) = \exp\left[-\sum_{m=1}^{k} \eta_m |x_m^i - x_m^j|^2\right] \tag{5}$$

where η_m $(\eta_m \geq 0,\ m = 1, \ldots, k)$ are unknown correlation parameters. The unknown parameters η_m must be obtained to produce a DACE model. In Eq. (5), x_m^i and x_m^j are the mth elements of point vectors $\mathbf{x}_i$ and $\mathbf{x}_j$. The value of the correlation function is the degree of influence between two points against the response. A higher value means a higher degree of influence; the surrounding points affect the point. Each design variable has a different scale for the correlation parameter. Therefore, the strength of the effect of the correlation parameter is controllable using the unknown value of η_m.

The expected response at a point vector $\mathbf{x}$ is calculable as

$$\hat{y}(\mathbf{x}) = \hat{\mu} + \mathbf{r}^T \mathbf{R}^{-1}(\mathbf{y} - \mathbf{1}\hat{\mu}) \tag{6}$$

where $\hat{\mu}$ is an estimator of μ, $\mathbf{y}$ is a column vector that has n elements of responses at n sample points, $\mathbf{1}$ is a column vector whose elements are all 1 $([1, \ldots, 1]^T)$, and $\mathbf{r}$ is a column vector that has values of the correlation function between the target point and the other sample points.

$$\mathbf{r}(\mathbf{x}) = [R(\mathbf{x}, \mathbf{x}^1), R(\mathbf{x}, \mathbf{x}^2), \ldots, R(\mathbf{x}, \mathbf{x}^n)]^T \tag{7}$$

Assuming that the values of $\boldsymbol{\eta}$, a parameter vector of the DACE model, are given, then $\hat{\mu}$ and the variance of $\hat{\mu}$ $(\hat{\sigma}^2)$ are calculable as the following:

$$\hat{\mu} = \frac{\mathbf{1}^T \mathbf{R}^{-1} \mathbf{y}}{\mathbf{1}^T \mathbf{R}^{-1} \mathbf{1}} \tag{8}$$

$$\hat{\sigma}^2 = \frac{(\mathbf{y} - \mathbf{1}\hat{\mu})^T \mathbf{R}^{-1} (\mathbf{y} - \mathbf{1}\hat{\mu})}{n} \tag{9}$$

The sole remaining problem is how to obtain the parameter vector $\boldsymbol{\eta}$. For the estimation of the values of $\boldsymbol{\eta}$, a log likelihood function is used here, as

$$Ln(\boldsymbol{\eta}) = -[n \ln(\hat{\sigma}^2) + \ln |\mathbf{R}|]/2 \tag{10}$$

where $\hat{\sigma}^2$ and $\mathbf{R}$ are functions only of $\boldsymbol{\eta}$. The correlation parameter for $\boldsymbol{\eta}$ is obtained using the maximization of the log likelihood function. In this problem, the number of unknown values is k. In this study, the particle swarm optimization (PSO) method [24] is used to maximize the log likelihood function with the initial values that Welch et al. proposed [25], determining the most influential parameters one by one while fixing the others. After determining the values using the same process, these values are used for the initial positions of entire particles in the PSO. Therefore, the PSO searches the optimal parameters around the provisional optimal result obtained with the method used by Welch et al. [25].

After obtaining the correlation parameter vector $\boldsymbol{\eta}$, the estimator of Eq. (6) is calculable and the variance of the estimator is obtained as

$$s^2(\mathbf{x}) = \hat{\sigma}^2 \left[1 - \mathbf{r}^T \mathbf{R}^{-1} \mathbf{r} + \frac{(1 - \mathbf{1}^T \mathbf{R}^{-1} \mathbf{r})^2}{\mathbf{1}^T \mathbf{R}^{-1} \mathbf{1}}\right] \tag{11}$$

The estimator of the standard deviation is calculated as $s = \sqrt{s^2(\mathbf{x})}$; its value is defined as an estimation error. The DACE model definitely passes all sampling points. The estimation errors at the sampling points are definitely zero; the estimation error becomes larger when the estimation point is located far from the adjacent sampling points.

When the estimation error is subjected to a normal distribution of $N(y, s^2)$, the probability that the response $y = f(\mathbf{x})$ is greater than a value of 1 is calculated as

$$\Pr(f(\mathbf{x}) \geq 1) = \Phi((f(\mathbf{x}) - 1)/s) \tag{12}$$

where $\Phi((f(\mathbf{x}) - 1)/s)$ is the cumulative distribution function, $f(\mathbf{x})$ is a response y of the kriging (DACE) model at position vector $\mathbf{x}$, and s is the estimator of the standard deviation obtained from Eq. (11). The kriging response surface is used only for the buckling load ratio constraint in this study. Therefore, $y = f(\mathbf{x}) > 1$ simply means that the probability of satisfaction of the buckling load constraint is greater than 50%. Consequently, the probability of satisfaction of the buckling load constraint is not zero even when $y = f(\mathbf{x}) < 1$. The probability might be a large value such as 30 or 40% even when $y = f(\mathbf{x}) < 1$ if the estimation error (the estimator of the standard deviation) is large.

We obtained a probability of satisfaction of the buckling load constraint. Using this probability, the optimization problem described in Eq. (1) can be replaced by finding the minimum weight design with a higher probability of satisfaction of the constraint in EGO. This can be written as

$$\text{maximize EI} = \max[W_{\min} - W(\mathbf{x}), 0] \times \Pr(f(\mathbf{x}) \geq 1) \tag{13}$$

where $W_{\min}$ is the minimum weight in the sampling structures that satisfies the constraint and is regarded as the provisional optimal result. Therefore, the value of $\max[W_{\min} - W(\mathbf{x}), 0]$ represents the improved decrease of weight. Furthermore, $\Pr(f(\mathbf{x}) \geq 1)$ shows the probability of satisfaction of the constraint. This objective function is designated herein as the EI. The weight is light and the probability of satisfaction of the constraint is high when the value of the EI is bigger. The value of the EI is therefore selected as the new objective function in the EGO. In a previous paper [21], the EI was maximized using two-level optimization with the PSO method; the kriging surrogate model was improved using the additional sampling points selected from the values of the EI. With the progress of the optimization iteration processes, it became difficult to obtain an optimal result that satisfied the buckling load constraint. That difficulty resulted from the fact that the EI was affected by the structure that had a large reduction of the weight with not such a higher probability of satisfaction of the buckling load constraint.

For this study, the value of the EI is separated into two objectives: weight reduction and the probability of satisfaction of the constraint. That separation enables us to distinguish the structures in the Pareto frontier: structures that have a higher probability of constraint and structures that exhibit a greater weight reduction.

B. Design of Experiments

Latin hypercube sampling (LHS) experiments were used for this study [20]. In fact, LHS yields a uniformly distributed sampling in the entire design space. Usually more than 10-times-larger samplings are required for the DACE model [24].

However, LHS is not simple for composite structures because the lamination parameters are not independent variables. All feasible symmetric laminates of 16 plies (the half-ply number is eight) were prepared to solve this problem. The number of feasible plies that satisfy the balanced rule of angle plies is 3281. From this set of feasible laminates, a set of n_s laminates is selected randomly with some constraints. The constraints that are described next are imposed on the in-plane lamination parameters.

For laminates of 16 plies, 25 sets of in-plane lamination parameters are feasible. As discussed, the available fiber angles are limited to 0, 45, −45, and 90 deg. For the feasible laminates, including these fiber angles, the lamination parameters exist inside and on the boundary of the triangle (1,1), (−1, 1), and (0, −1)[9]. The three apices of the triangle are selected at least once and another 22 points are selected at least twice to obtain an equal distribution of sampling for the in-plane lamination parameters. The selected laminates are feasible even when the number of plies is changed by changing the ply thickness. For example, let us consider the case of a laminate of $[0_2/45/-45/90/45/-45/90]_s$. The same set of lamination parameters is obtainable by doubling each ply as $[0_4/45_2/-45_2/90_2/45_2/-45_2/90_2]_s$ when the total number of plies is changed to 32.

All variables are divided into n_s segments to select candidate points for variables of dimensions. For the variables of dimensions of structural components such as a hat-stiffener configuration, feasible design scopes of the variable are normalized from −1 to +1 and equal segmentations are used here. For the number of plies N, the numbers must be positive integers. This is achieved using rounded numbers. From these segmented variables, a table of all segments is made and equally distributed sampling points are obtained.

After obtaining the sampling points, FEM analyses are performed at all sampling points. Using the results of the FEM analyses, a kriging model of the buckling load ratio is constructed. In this study, $n_s = 251$ is used.

C. Optimization Using the Multiple-Objective Genetic Algorithm

The MOGA algorithm has already been published in a well-known textbook [26]. The MOGA process resembles that of the simple genetic algorithm (GA). Therefore, a detailed explanation is omitted from this discussion. Two points distinguish it from the simple GA: the objective functions are multiple and they are not transformed into a single objective function. The population of the MOGA is scattered over a wide range to obtain nondominated solutions (Pareto frontier solutions) distributed in the entire design region (the population must be rich in diversity). The definitions of these differences are described in the literature [26].

In the present study, population diversity is attained using the fitness shearing method [26]. First, the method calculates the distance (d) between two individuals and evaluates the degree of similarity between them with a shearing function $Sh(d)$, which is a concentric ring of contour lines. Based on the degree of similarity, a modification coefficient is multiplied to the fitness function of the target individual. The fitness function is reduced by the modification coefficient when the degree of similarity is high.

This study included 100 individuals; the terminal generation was 300. For selection, a Pareto ranking method was used: a randomly selected double-point crossover of the possibility 0.9 was used here for bit-type variables, and a simulated binary crossover of the possibility of 0.5 was used for real-number variables. For mutation of the binary variables, the uniform mutation rate was set to 0.1 and a polynomial mutation of rate of 0.5 was applied for the real-number variables. A well-known Pareto ranking method was used for evaluations; the niche coefficient of the fitness shearing was 0.2. These values were determined using a trial and error.

As described previously, this study deals with the EI in Eq. (13), but the EI is separated into two objectives: the structural weight reduction of the hat-stiffened panel and the probability of satisfaction of the buckling load constraint. Therefore, the problem becomes one of maximization of the two objectives: the reduction of the weight from the provisional optimal and the probability of satisfaction of the buckling load constraint. Recalling that the buckling load ratio is approximated here and that the kriging model response is a simple approximated value, the probability of satisfaction is used rather than the direct response of the kriging model.

This modified method can be compared with the normal EGO using the EI. Figure 2 depicts a schematic representation of the Pareto frontier of this method. The abscissa shows the probability of satisfaction of the constraint and the ordinate is the weight reduction. The value of the EI in Eq. (13) is the multiple product of the abscissa and the ordinate. Figure 2 shows that the normal EI is expressed by the inversely proportional curve. Considering that the Pareto frontier of this method is obtained as presented in Fig. 2, the maximization of the EI is the same as finding the cross point between the Pareto frontier and the inversely proportional curve of the normal EI. As presented in Fig. 2, the Pareto frontier includes the larger weight reduction structures in region II and the higher probability of satisfaction of the constraint in region I. The maximization of the EI finds a slightly larger reduction of the weight and a slightly lower possibility of the satisfaction of the constraint. The obtained cross point usually does not satisfy the constraint when the buckling load surrogate model is not sufficiently exact. Separation of the EI into the two objectives enables us to find the Pareto frontier, which also enables us to select appropriate structures to improve the buckling load constraint from the structures of higher possibility of satisfaction of the constraint with appropriate spacing between the sampling points. This point is the most important modification for the normal EGO in this study.

This study uses a two-layer optimization process that is identical to that of a method used with PSO [21]. In the upper layer, the optimization of structural dimensions is performed using MOGA. Chromosomes of each correspond to structural dimensions such as the height or width of the stiffener; these dimensions are sufficient information to evaluate the structural weight. In this study, the quantities of plies are positive integer numbers. The other dimensions and lamination parameters are real numbers. For this reason, we selected MOGA over other mathematical programming methods.

To evaluate the buckling load, however, stacking sequences of composite laminates must be known. The upper-layer optimization process activates a lower-layer optimization of the stacking

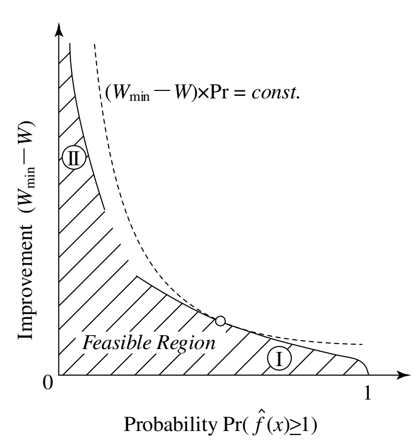

Fig. 2 Schematic representation of the Pareto frontier and curve of the normal EI method.

sequences for this process. The stacking sequences of the stiffener and panel are optimized using the modified FBB method, which optimizes more than two laminates simultaneously. These optimization processes are described as follows.

1) Structural optimization (upper-layer optimization):

The structural optimization process deals with all dimensions ($\boldsymbol{d}$) of a target composite structure, such as height or width of the stiffener and number of plies. The two objective functions are a reduction in weight from the provisional optimal structure and the probability of satisfaction of the buckling load constraint.

$$\text{maximize} \quad \max[W_{\min} - W(\mathbf{d})] \tag{14}$$

$$\text{maximize} \quad \max_{\mathbf{V}_p\mathbf{W}_p\mathbf{V}_s\mathbf{W}_s}[\Pr(\mathbf{d}, \mathbf{V}_p, \mathbf{W}_p, \mathbf{V}_s, \mathbf{W}_s)] \tag{15}$$

Of those expressions, Eq. (14) is the maximization of the reduction in weight from the provisional optimal structure $W_{\min}$; Eq. (15) is the maximization of the probability of satisfaction of the buckling load constraint. Only for Eq. (15) are lamination parameters of the stiffener and panel listed as design variables in addition to dimensions $\boldsymbol{d}$. The process is performed using the genetic algorithm. Consequently, each has already fixed values of dimensions for the structure before calculation of the evaluation of fitness; the dimensions are fixed before a calculation of the buckling load. The buckling load depends on the stacking sequences of the stiffener and panel. Therefore, the upper-layer process activates the lower-layer process for the maximum buckling load with the dimensions and changing stacking sequences.

2) Multiple-stacking-sequence optimization (lower-layer optimization):

The lower-layer optimization closely resembles a subroutine call in a software program. The lower-layer optimization is applied every time the upper-layer optimization requires evaluation of the buckling load to decide stacking sequences of the stiffener and panel. In this optimization, the dimensions (d) of the carbon-fiber-reinforced plastic (CFRP) structure are values given by the upper-layer optimization. Here, the stacking sequences of two laminates (stiffener and panel) are the design variables. Lamination parameters of the panel ($\mathbf{V_p}, \mathbf{W_p}$) and stiffener ($\mathbf{V_s}, \mathbf{W_s}$) are treated as variables instead of the direct fiber angle sets.

$$\text{maximize} \quad \frac{f(\mathbf{d}, \mathbf{V}_p, \mathbf{W}_p, \mathbf{V}_s, \mathbf{W}_s) - 1}{s} \quad \text{subject to } \mathbf{d} = \text{const.} \tag{16}$$

The objective function of Eq. (16) is equal to the variation of the cumulative distribution function of Eq. (12). The cumulative distribution function is a monotonically increasing function. Therefore, the increase in the value of Eq. (16) is equal to the increase in the probability of satisfaction of the buckling load with changing stacking sequences of the stiffener and panel. For optimization of the stacking sequences for multiple laminates, the new iteration FBB method is used [16].

For the FBB method, however, the objective function must be approximated using a quadratic polynomial. The D-optimal stacking sequences of feasible laminates are selected, and the buckling load values are calculated using the kriging model to produce an objective function from the quadratic polynomial. The results are approximated using the quadratic polynomial with the least-squares-error method [14]. The maximum buckling load ratio when all the dimensions are fixed and the stacking sequences are the design variables is provided to the upper-layer optimizations.

The flow of this optimization process is summarized as follows and as depicted in Fig. 3.

1) First, the design of experiments is performed using LHS to select n_s points. In fact, FEM analyses of buckling load are conducted at the selected n_s points. The kriging model is produced from the results of the n_s points. The number of n_s points is expected to be greater than 10 times the number of design variables. From the selected points n_s, the minimum weight structure that satisfies the buckling load constraint is selected as the provisional optimal structure ($W_{\min}$).

2) Initial individuals of the MOGA (total population is 100) are selected randomly.

3) Upper-layer optimization is performed using the MOGA.

a) The fitness of each is evaluated; lower-layer optimization is performed to optimize stacking sequences of the stiffener and panel using the modified FBB method.

b) Nondominated individuals are sought using the two objective functions: weight reduction from the provisional structure and the probability of satisfaction of the buckling load.

c) The population diversity is evaluated for each; the fitness value of each is calculated.

d) Selection is performed using a Pareto ranking method; a mutation is conducted after a crossover of the selected individuals.

4) The MOGA is terminated at the 300th generation.

5) The top 10 individuals with the probability of satisfaction of the buckling load are calculated using FEM while considering the diversity of the population in the palette of 100 solutions.

The 10 new FEM results are added to the kriging model data set. This process is cycled from step 2.

Two-layer optimization with the FBB is used here instead of optimizing the stacking sequence in the upper layer together with the dimensions of the stiffened panel because of the computational cost and the reliability of optimizations of a stacking sequence. To find an optimal stacking sequence using the GA is not reliable: it required tens of runs of the GA, as described in a previous paper [27]. In contrast, the FBB is a deterministic efficient method based on a branch-and-bound method.

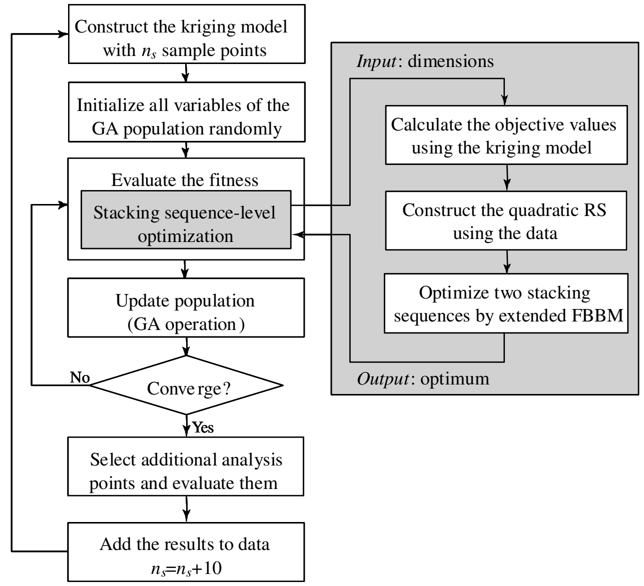

Fig. 3 Flowchart of multi-objective dimensions and stacking sequence optimization.

IV. Results and Discussion

A selection of 251 points was made using the LHS experiment design and FEM analyses conducted at the selected sampling points. The calculated initial kriging model parameters are presented in Table 3. Cross-validations of the kriging model were performed at all points; the results are portrayed in Fig. 4. Cross-validation is the method of checking the kriging model. A kriging model is calculated from a set of sampling data except for a selected point. An estimation of the selected point is made using the calculated kriging model. An estimation of the kriging model is then performed at the selected point. Therefore, all data portrayed in Fig. 4 are new data for the

Table 3 Estimated model parameters of the initial kriging RS

Design variable	Model parameter η_m
V_{1p}^*	0.286
V_{2p}^*	0.0181
W_{1p}^*	0.109
W_{2p}^*	0.0647
V_{1s}^*	0.459
V_{2s}^*	0.190
W_{1s}^*	0.0336
W_{2s}^*	3.04e − 7
h	0.313
w	9.42e − 4
w_2	0.190
N_p	0.478
N_s	0.190
$(b_2 - w_2)$	0.0295

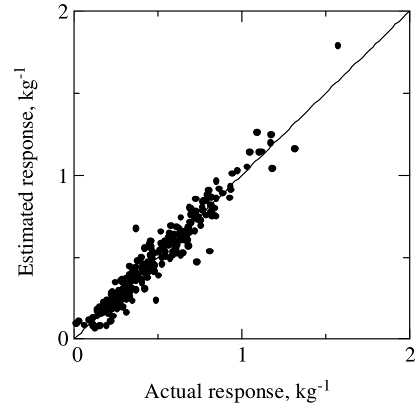

Fig. 4 Result of the cross-validation of the initial RS.

kriging model. The abscissa is the true value and the ordinate is the estimated value. Good estimations are plotted on the diagonal line. Figure 4 shows that the kriging model provides good estimations for almost all sampling points.

Using this initial kriging model, the first optimization run of the MOGA is performed for up to 50 generations. An extra 10 FEM analyses were added to the kriging model data set to improve the kriging model after each run. Using the improved kriging model, five runs of the MOGA were performed. Figure 5 shows the results of the minimum weight history of each run of the MOGA. The ordinate is the weight of the structure and the abscissa is the run number of the MOGA. The result for the zero cycle is the provisional optimal structure obtained from the initial set selected from LHS. The open circle symbols are the minimum weight design that satisfies the constraint at each run; the solid circle symbols show the results that satisfy the constraint but that have heavier weight, and the cross symbols show results that do not satisfy the constraints. As presented in Fig. 5, the results of each run obtained from the MOGA always include feasible solutions that satisfy the constraint of the buckling load because the structures that have a high probability of satisfaction of the buckling load are always selected.

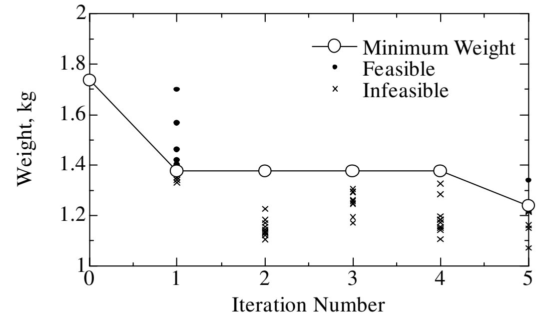

Fig. 5 Iteration history of the minimum weight for the MOGA.

The obtained 100 Pareto frontier solutions after the first runs (50 generations) are depicted in Fig. 6. The abscissa is the probability of satisfaction of the buckling load; the ordinate is the reduction in the weight. As described earlier, the product of the abscissa and the ordinate is the EI. The small solid circles show the palette of solutions; the 10 larger symbols represent solutions added to the data set for remaking the kriging model. To improve the kriging model, the separation of the EI into the two objectives enables us to select appropriate candidates: structures having a higher probability of satisfaction of the constraint and similar structures judged using the fitness shearing used in the MOGA are excluded. After selection, FEM analyses were performed. The solid large circles show structures that do not satisfy the buckling load constraint. The open large circles are structures that satisfy the buckling load constraint according to the results of the FEM analyses. This figure shows that

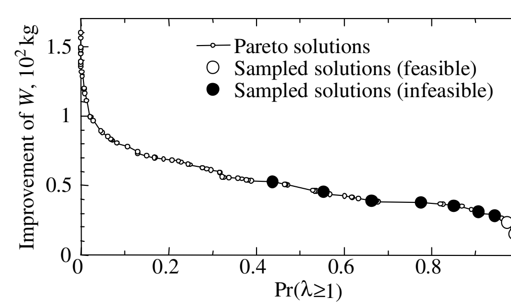

Fig. 6 Example of Pareto solutions and sampled solutions.

the structures that have a probability of satisfaction of the constraint of higher than 95% practically satisfy the constraint. Using the same process of fitness sharing, the structures that have a higher reduction of weight are also selected.

Table 4 presents the optimal solution obtained after five cycles of optimizations of the MOGA. In optimizations of the MOGA, the FEM analyses were 301. The obtained minimum weight was

Table 4 Optimal design using the multi-objective method

W, kg	λ	h, m	w, m	W_2, m	$(b_2 - w_2)$, m	Plies	Stacking sequence
1.239	1.01	0.0235	0.0432	0.103	0.040	N_p: 6, N_s: 10	$[(\pm 45)_3]_s$, $[0_2/45/0_4/-45/0_2]_s$

Table 5 True optimal design

W, kg	λ	h, m	w, m	W_2, m	$(b_2 - w_2)$, m	Plies	Stacking sequence
1.203	1.00	0.022	0.038	0.105	0.050	N_p: 6, N_s: 9	$[(\pm 45)_3]_s$, $[0/45/0_4/-45/0_2]_s$

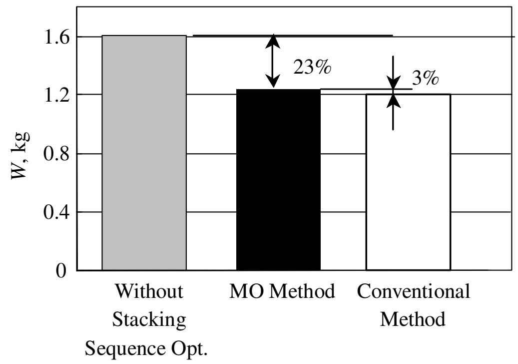

Fig. 7 Comparison of optimal results.

1.239 kg. Figure 7 portrays a comparison of the optimal designs. The result without stacking sequence optimization is the minimum weight optimized fixed stacking sequence of quasi-isotropic laminates. The dimensions were selected as design variables in this case. The result obtained using the conventional method (see Table 5) is the optimal result obtained from tens of thousands of random FEM analyses. The result obtained from the MOGA is only 3% heavier than the true optimal result and 23% lighter than the result without stacking sequence optimizations.

The 3% error compared with the true optimal comes from the kriging surrogate model. A surrogate model usually uses a smooth surface model. A previous study, using a genetic algorithm with quadratic polynomials as a surrogate model, also showed the error arising from the approximation error of the polyhedral buckling load ratio caused by buckling mode changes with a smooth model [28]. More runs of the MOGA with improvement of the kriging model are necessary to prevent the error.

V. Conclusions

For optimization of the dimensions of a stiffened panel and stacking sequences, a new optimization process is proposed. The process, which is classified into two layers, uses MOGA for the optimization of the dimensions of the stiffened panel and the modified FBB method for the evaluation of individuals of the MOGA to obtain the maximum performance of the stacking sequences. The expected improvement is separated into two objectives: weight reduction and the probability of satisfaction of the buckling load constraint. This separation of the expected improvement into two objectives enables us to select appropriate candidates that have a higher probability of constraints and to thereby improve the kriging model. This modification of the efficient global optimization method is applied to a dimension-and-laminate optimization problem of a hat-stiffened composite panel. Consequently, a practically optimal result was obtained using only 301 FEM analyses with a 3% error from the true optimal structure.

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E. Livne
Associate Editor