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Extended fractal branch and bound method for optimization of multiple stacking sequences of stiffened composite panel *

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Abstract—Stiffened composite panels are often used as structural components in aircraft in order to avoid buckling. It is well known that stacking sequence optimizations are indispensable for laminated composite structures. Stiffened composite panels usually have more than two stacking sequences because they consist of a panel skin laminate and stiffener laminates. This means that the stacking sequences need to be jointly optimized to achieve structural optimization of the stiffened composite panel. The authors have proposed a new stacking sequence optimization method, called the fractal branch and bound method, for optimizing a single laminate. In the present study, the fractal branch and bound method is extended to optimizing multiple stacking sequences. The extended method is applied for obtaining two optimal stacking sequences for the maximization of the buckling load of a hat-stiffened composite panel. The improved method successfully provides two optimal stacking sequences determined in a short period of time.

Keywords: Optimization; buckling; stacking sequence; laminates.

1. INTRODUCTION

Laminated composites are usually made by stacking unidirectional fiber prepreg sheets in various orientations. Laminated composites have superior specific stiffnesses and specific strengths relative to conventional metallic materials. This results in widespread applications for laminated composites for aerospace structures. The laminated composite structures usually adopt thin shell-type structures. For

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thin laminates, buckling failure is one of the main issues for practical designing. To improve the buckling load, thin laminated structures employ stiffeners such as the hat type or blade type.

For aerospace applications made of laminated composites, the optimizations of the stacking sequences are indispensable. Miki [1] and Fukunaga and Vanderplaats [2] have proposed a superior graphical optimization method by means of lamination parameters. For practical laminates, the available fiber angles are limited due to the lack of experimental data and the constraint of a hand lay-up fabrication process. Moreover, several constraints on the fiber angles exist due to empirical rules, such as the four-contiguous-ply rule for the prevention of large matrix cracking. As a result, the optimizations of stacking sequences become combinatorial optimization problems with combinatorial constraints. For practical stacking sequence optimizations, more than two laminates need to be optimized because practical aerospace structural components usually consist of panels and stiffeners made from composite laminates. Since both the stacking sequences of the skin panels and that of the stiffeners affect the buckling load of the stiffened panel, the optimization of both of the laminates must be performed simultaneously.

Genetic algorithms (GAs) are adopted for the optimizations of stacking sequences [3]. Since the use of GAs is a stochastic search method, a GA requires a significant amount of parameter tuning processing to prevent a decrease in its performance. It is not easy to implement constraints into the algorithms of GAs. The authors have proposed an improved method for the implementation of the combinatorial constraints of the stacking sequences [4–6]. The evaluation of GAs is also computationally demanding. To reduce the high cost associated with evaluating the constraints, the author's group has proposed an approximation method by means of a response surface using lamination parameters [7, 8].

The authors' group has proposed a fractal branch and bound method for stacking optimization [9, 10], in addition to their work on GAs. The method employs quadratic polynomials for the response surfaces using the lamination parameters to approximate objective functions such as buckling loads. The method is not computationally intensive and the optimal results can be obtained by means of deterministic processes. The method is based on the discovery that plots of feasible laminates result in fractal patterns in the lamination-parameter space. Since the method is one of the branch and bound methods, parameter tuning is not required [11]. The method has been successfully applied to the problem of maximizing the buckling load of laminates [10], and the problem of maximizing the flutter limit [12, 13] with the constraints. A limitation of the method is its single ply optimization.

Therefore, in the present study, the fractal branch and bound method is improved for optimizations of more than two laminates, such as simultaneous panels and stiffeners. In the present method, a quadratic polynomial objective function is adopted with the variables of the lamination parameters of the two laminates: the stiffeners and the panel. The improved method is applied as a feasibility demonstration to the buckling load maximization problem of a stiffened composite panel with hat stiffeners.

2. OPTIMIZATION PROBLEM

The present study adopts the buckling load maximization problem of a hat-stiffened panel of composite laminates as a feasibility demonstration, as shown in Fig. 1. The component comprises a panel skin and two stiffeners. The panel and the stiffeners are made from laminated composites, and each has an independent stacking sequence of 16 plies. The boundary conditions of the problem are the same as those of the analysis of Bushnell and Bushnell [14]. The boundary condition of the loaded edges is in a fixed grip condition. The other two edges of the panel are free, except that the rotation around the x-axis is fixed to zero. This means that the stiffened panel is very large in the y-axis direction: the large panel is made from repetitions of this panel with the two stiffeners. A compression load of λN_x is applied in the x-axis at the edges. The material adopted here is T300/5208. All of the dimensions of the stiffened panel and the material properties are as follows:

$$a = 1\text{ m},\quad b = 0.25\text{ m},\quad b_2 = 0.16\text{ m},\quad w = 0.05\text{ m},\quad w_2 = 0.11\text{ m},$$
$$h = 0.05\text{ m},\quad E_x = 181\text{ GPa},\quad E_y = 10.3\text{ GPa},\quad G_{xy} = 7.17\text{ GPa},$$
$$\nu_{xy} = 0.28.$$

Each ply has a thickness of 0.125 mm.

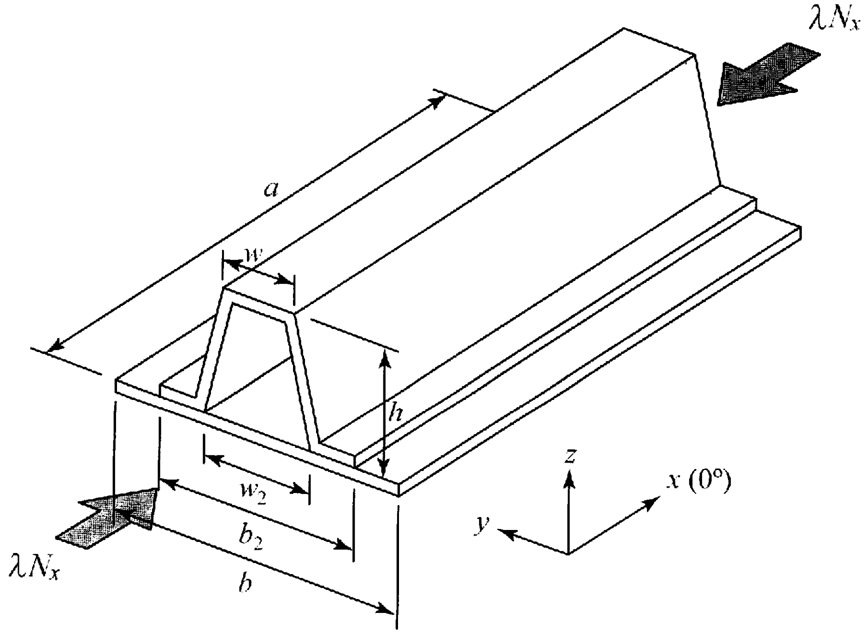


Figure 1. Analysis model of a hat-stiffened panel.

The dimensions such as the thickness of the panel and the spacing of the stiffeners have a large influence on the analysis of the buckling load of the hat-stiffened panel. These dimensions are, therefore, very important target parameters for practical optimal designs. In the present study, these parameters about the dimension of the hat-stiffened panel are all fixed to simplify the problem, because the objective of the present study is to demonstrate the effectiveness of the proposed method for stacking sequence optimizations. Only the interaction of the two kinds of stacking sequences of the laminates is dealt with here. Vitali *et al.* have optimized these dimensions of the hat-stiffened panel [15]. The present study adopts the same dimensions of the hat-stiffened panel as shown in the research of Vitali *et al.* As the loading condition of the present study is different from the research of Vitali *et al.*, these dimensions are not the exact optimum but are used as reference data for the maximization of the buckling load. For the practical optimizations of the maximization of the buckling load, these dimensions must be optimized simultaneously with the two stacking sequences of the panel and stiffeners. This is the subject of our future work. The number of stacks is also fixed here to 16 plies.

The computed buckling load is expressed as a scaling factor against the reference load $N_x = 100$ kN/m. The minimum buckling load is used as the buckling load factor λ. In the present study, the commercially available FEM code ANSYS is employed for the buckling load analysis. Laminated shell elements are used in the analysis. The total number of elements is 2800 and the total number of grids is 8858.

As previously mentioned, due to manufacturing limitations and the lack of experimental data, the available fiber angles are limited to 0°, ±45°, and 90°. Moreover, the following additional constraints are implemented:

(1) No more than five contiguous plies are allowed to be at the same fiber angle (four-ply rule).

(2) The number of −45°- and +45°-plies must be balanced (balance rule).

The first constraint is implemented to prevent large matrix cracking, and the second is intended to avoid shear-tensional coupling and to reduce tension-twisting coupling.

With these constraints, the stacking sequence optimization of panel skin and stiffener for buckling load maximization is the target of the present study.

3. LAMINATION PARAMETERS

In-plane and out-of-plane stiffnesses of symmetric laminates are expressed by means of lamination parameters. A symmetric laminate has four in-plane lamination parameters (V_1^*, V_2^*, V_3^* and V_4^*), and four out-of-plane lamination parameters (W_1^*, W_2^*, W_3^* and W_4^*). In the present study, as the available fiber angles are limited, the V_3^*, V_4^* and W_4^* vanish and the W_3^* is a small value due to the balance rule. Thus, we deal with the rest of the four lamination parameters (V_1^*, V_2^*, W_1^*, W_2^*) for each laminate.

The in-plane and out-of-plane lamination parameters of the symmetric laminates of $2N$ plies (total number of plies is $2N$) are defined as follows:

$$\mathbf{V} = \begin{bmatrix} V_1^* \\ V_2^* \end{bmatrix} = \sum_{k=1}^{N} \left(a_{k-1}^V - a_k^V\right) \begin{bmatrix} \cos 2\theta_k \\ \cos 4\theta_k \end{bmatrix},$$
$$a_k^V = \frac{N-k}{N}, \tag{1}$$

$$\mathbf{W} = \begin{bmatrix} W_1^* \\ W_2^* \end{bmatrix} = \sum_{k=1}^{N} \left(a_{k-1}^W - a_k^W\right) \begin{bmatrix} \cos 2\theta_k \\ \cos 4\theta_k \end{bmatrix},$$
$$a_k^W = \left(\frac{N-k}{N}\right)^3, \tag{2}$$

where θ_k is the fiber angle of the kth ply from the outermost ply ($k = 1 \ldots N$).

For the stacking sequence optimization of the present study, eight lamination parameters of both the skin panel and the stiffeners are dealt with. The lamination parameters of the panel are expressed as V_{1p}^*, V_{2p}^*, W_{1p}^*, and W_{2p}^*, and the lamination parameters of the stiffener are expressed as V_{1s}^*, V_{2s}^*, W_{1s}^*, and W_{2s}^*.

4. RESPONSE SURFACE METHOD

The purpose of the response surface method is to obtain a formula for the relationship between the explanatory variables of x_i ($i = 1 \ldots M$, M is the number of explanatory variables) and the response y. For the present study, a quadratic polynomial is adopted as follows:

$$y = \beta_0 + \sum_{i=1}^{M} \beta_i x_i + \sum_{i=1}^{M} \sum_{j \geqslant i}^{M} \beta_{ij} x_i x_j, \tag{3}$$

where β are the unknown coefficients determined by means of the least square errors method. In the present study, the explanatory variables are the eight lamination parameters of the panel and the stiffener. For the response surface, the lamination parameters are replaced by the new variables ($x_1 = V_{1p}^*$, $x_2 = V_{2p}^*$, $x_3 = W_{1p}^*$, $x_4 = W_{2p}^*$, $x_5 = V_{1s}^*$, $x_6 = V_{2s}^*$, $x_7 = W_{1s}^*$, and $x_8 = W_{2s}^*$) and the response is the objective function y ($= \lambda$).

To obtain the unknown coefficients by means of the least square errors method, sets of responses of several different sets of variables are prepared using experiments or computations. These sets are called sample data. The total number of sample data points must be more than the total number of unknown coefficients. Generally, an experimental design is adopted which reduces the variance of the coefficients. In the present study, D-optimal sets are selected from the feasible laminates without considering the two constraint rules for stacking sequences. The selection of the D-optimal laminates is explained in detail in reference [7].

5. FRACTAL BRANCH AND BOUND METHOD

The fractal branch and bound method was proposed by the authors' research group as a fast and deterministic optimal stacking sequence searching method [9, 10]. The method requires an approximation of the design space using a quadratic polynomial of the lamination parameters as explanatory variables for the response surface. The method provides the deterministic optimal stacking sequence in a short period of time but the method is limited to a single laminate. For optimizations of multiple laminates considering the interaction between multiple laminates was not allowed in the previous studies. This limitation is overcome in the present study.

Let us consider the case where the response surface of the approximated objective function is obtained for a single laminate. In this formula, V and W are the vectors comprising the in-plane and out of plane lamination parameters, respectively.

$$f = f(\mathbf{V}, \mathbf{W}). \quad (4)$$

The optimal stacking sequence is the feasible laminate that gives the maximum response y. The stacking sequence of each laminate makes a tree-structure chart as shown in Fig. 2 in search of the fiber angles of the outer plies. Since there is no difference in terms of the lamination parameters if we exchange the 45°-plies with −45°-plies, all of these angle plies are expressed as 45°-plies in the tree structure. The fractal branch and bound method is a simple branch and bound method for searching for the optimal stacking sequence. For fast searching, an efficient pruning method for searching the tree branch is indispensable. The fractal branch and bound method uses fractal patterns that are obtained when we plot all of the feasible

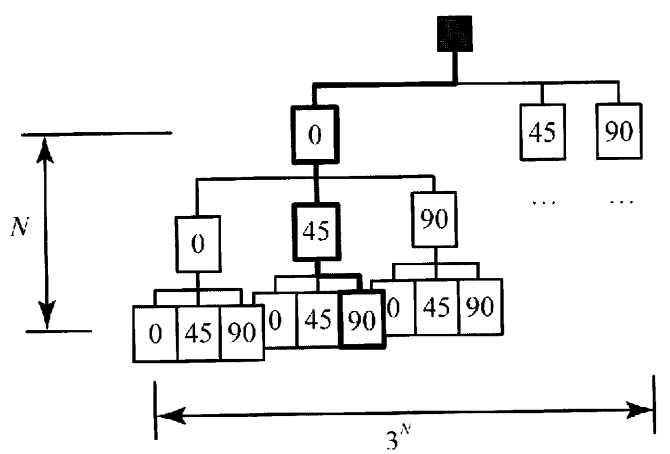


Figure 2. Tree structure of a stacking sequence.

laminates in the lamination parameter space for shrinking the search branches. That enables us to optimize in a short amount of time.

5.1. Fractal pattern made from lamination parameters

The values of the trigonometric functions used in the lamination parameters in equations (1) and (2) are limited to the cases expressed as follows due to the constraints.

$$\begin{bmatrix}\cos 2\theta_k\\ \cos 4\theta_k\end{bmatrix} = \overset{0^\circ}{\begin{bmatrix}1\\1\end{bmatrix}}\ \overset{\pm 45^\circ}{\begin{bmatrix}0\\-1\end{bmatrix}}\ \overset{90^\circ}{\begin{bmatrix}-1\\1\end{bmatrix}}. \tag{5}$$

We can consider the lamination parameters as the elements of the in-plane and out-of-plane lamination parameter vectors V and W. The coefficients of the vectors in equations (1) and (2) are all positive values and the total sum of the coefficient satisfies the equations as follows.

$$\sum_{k=1}^{N}(a_{k-1}^{V} - a_k^{V}) = 1, \quad \sum_{k=1}^{N}(a_{k-1}^{W} - a_k^{W}) = 1. \tag{6}$$

These equations mean that the values of the lamination parameter are limited to the boundary and the inside of the triangular region of which the tree vectors in equation (3) represent the tree apices.

Next, we consider the case where the outer d plies are chosen and the inner plies are unknown. This laminate can be expressed as $[\theta_1/\theta_2/\cdots/\theta_d/*/*/\cdots/*]_S$. Let us consider the area in the lamination parameter space in which these laminates exist. The lamination parameters can be expressed as follows.

$$\mathbf{V} = \mathbf{V}_0 + a_d^{V}\mathbf{V}', \quad \mathbf{W} = \mathbf{W}_0 + a_d^{W}\mathbf{W}', \tag{7}$$

$$\begin{aligned}\mathbf{V}_0 &= \sum_{k=1}^{d}(a_{k-1}^{V} - a_k^{V})\begin{bmatrix}\cos 2\theta_k\\ \cos 4\theta_k\end{bmatrix},\\ \mathbf{W}_0 &= \sum_{k=1}^{d}(a_{k-1}^{W} - a_k^{W})\begin{bmatrix}\cos 2\theta_k\\ \cos 4\theta_k\end{bmatrix},\end{aligned} \tag{8}$$

$$\begin{aligned}\mathbf{V}' &= \frac{1}{a_d^{V}}\sum_{k=d+1}^{N}(a_{k-1}^{V} - a_k^{V})\begin{bmatrix}\cos 2\theta_k\\ \cos 4\theta_k\end{bmatrix},\\ \mathbf{W}' &= \frac{1}{a_d^{W}}\sum_{k=d+1}^{N}(a_{k-1}^{W} - a_k^{W})\begin{bmatrix}\cos 2\theta_k\\ \cos 4\theta_k\end{bmatrix}.\end{aligned} \tag{9}$$

Since $\mathbf{V}_0$ and $\mathbf{W}_0$ in equation (8) depend only on the decided fiber angles, these values can be obtained uniquely. The fiber angles in $\mathbf{V}'$ and $\mathbf{W}'$ of equation (9) are not chosen. In equation (9), all of the coefficients are positive values and the total sum of the coefficients is equal to unity. This means that the feasible values of $\mathbf{V}'$ and $\mathbf{W}'$ of equation (9) are limited to the inside of and on the edge of the triangle constructed from the three vectors defined in equation (5). By definition, the coefficients of a_d^V and a_d^W are positive values and are all smaller than unity. This implies that the feasible laminates are located on the edge of and on the inside of a new triangle whose center moves to $(\mathbf{V}_0, \mathbf{W}_0)$ in the lamination parameter space, and the length of the sides of the triangle is shrunk by the factor of a_d^V and a_d^W, respectively.

As the number of fixed plies d increases, the coefficients a_d^V and a_d^W decrease. The decrease in a_d^V and a_d^W means a shrinkage in the self-similar triangle. This process results in a fractal pattern of the feasible laminates in the lamination parameter space, as shown in Fig. 3. The fractal pattern of Fig. 3 is made by plotting all of the laminates of the 12-ply symmetric laminates. In Fig. 3, the small triangle that exists inside the large triangle represents the domain of the set of laminates that are represented by means of the stacking sequence of $[0/45/90/*/*/*]_S$.

5.2. Evaluation function

The mechanism for creating the fractal patterns described in the previous section brings about the branch and bound method. Let us consider the case where the ply angles of the outer d plies have already been decided again: the stacking sequence is given by $[\theta_1/\theta_2/\cdots/\theta_d/*/*/\cdots/*]_S$. The objective function is an approximate function made by means of the response surface of a quadratic polynomial. The variables of the objective function are the lamination parameters here. The objective function is expressed by substituting equation (7) as follows.

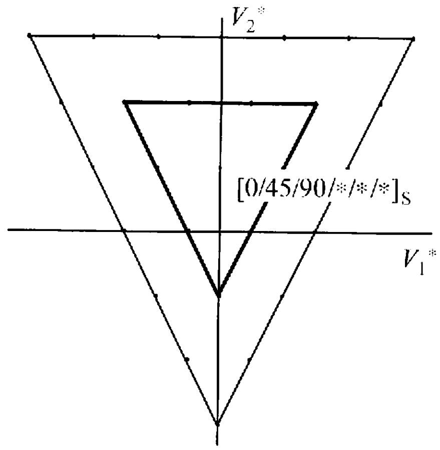
$$\begin{aligned} f &= f(\mathbf{V}, \mathbf{W}) = f\left(\mathbf{V}_0 + a_d^V \mathbf{V}', \mathbf{W}_0 + a_d^W \mathbf{W}'\right) \\ &= f_0 + f_{V'} + f_{W'} + f_{V'W'}, \end{aligned} \tag{10}$$

where f_0 is a fixed value calculated by the outer fixed ply angles, $f_{V'}$ is a quadratic polynomial comprising $\mathbf{V}'$, $f_{W'}$ is a quadratic polynomial comprising $\mathbf{W}'$, and $f_{V'W'}$ is a linear equation comprising the interaction term of $\mathbf{V}'$ and $\mathbf{W}'$. A detailed explanation is given in reference [10].

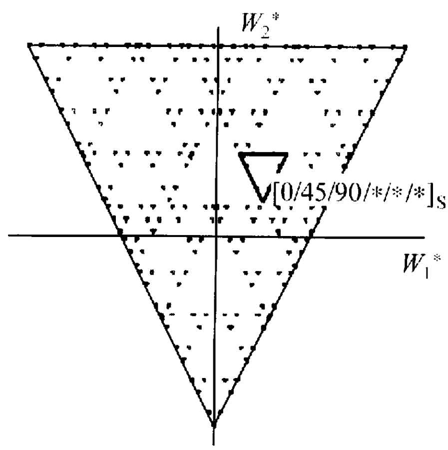
To obtain a conservative evaluation of the maximum value of a searching laminate for the branch and bound method, we need to evaluate the function g that gives a conservative maximum value of the target laminate. In the present study, a conservative evaluation means being equal to or greater than the real values of all of the possible laminates for the target laminates (the panel and stiffeners).

$$g = f_0 + \max\left(f_{V'}\right) + \max\left(f_{W'}\right) + \max\left(f_{V'W'}\right). \tag{11}$$

Although practical lamination parameters are discrete values, we assume that the lamination parameters are continuous variables to simplify the estimation of equation (11). For the estimations of the values of g in equation (11), we can assume



a) In-plane lamination parameter coordinates



b) Out-of-plane lamination parameter coordinates

Figure 3. Fractal pattern drawn by plotting all of the feasible laminates of 12 plies ($N = 6$).

that the lamination parameters $\mathbf{V}'$ and $\mathbf{W}'$ are independent continuous variables. These simplifications do not violate the conservative evaluation.

Since the lamination parameters $\mathbf{V}'$ and $\mathbf{W}'$ are correlated discrete valuables for the practical laminates, the values of g are equal to or greater than the values of f at any of the laminates. This means that the evaluation function g gives a conservative estimation at any of the laminates: the value of g is always equal to or greater than the values of any of the feasible laminates. Since $f_{V'}$ and $f_{W'}$ are simple quadratic polynomials of a single variable, the maximum values can be easily estimated. The values of $f_{V'W'}$ are always located at the apexes of the triangular

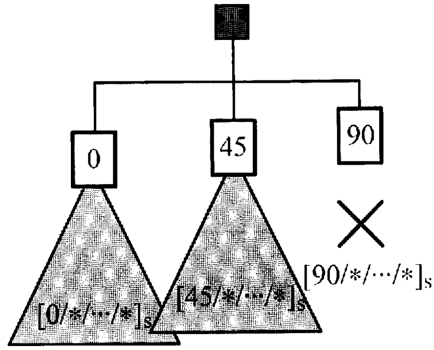


Figure 4. Pruning process for stacking the sequence optimization.

region because it comprises only of the linear interaction of $\mathbf{V}'$ and $\mathbf{W}'$. The values of g in equation (11) are, therefore, easily able to be calculated.

5.3. Branch and bound method

Searching in the fractal branch and bound method starts from the outermost ply, and progresses to the inner plies (branch of the tree structure). In the process of searching, the possibility of the existence of a superior laminate to the provisional laminate is checked by means of the value of the estimation function g. If the estimation g tells us that there is no superior laminate in the searching branch (a set of laminates for which the ply angles of the outer plies are decided and the inner plies are not decided), we can prune the branch (bound process) from the list of candidates. For example, let us consider the case where only the outermost ply has been decided, for example $[\theta_1/*/\cdots/*]$s. When we estimate the branch of laminates that start from the 90°-ply, expressed as $[90/*/\cdots/*]$s, we estimate the value of the estimation function g. If the value of g is less than the value of f of the provisional laminate in the triangular region of the set of laminates of $[90/*/\cdots/*]$s, we can eliminate the branch of $[90/*/\cdots/*]$s: further searches of the laminates such as $[90/45/\ldots/*]$s or $[90/0/\cdots/*]$s are not performed because there is surely no superior laminate in the set, as shown in Fig. 4. In the same way, we can prune the useless branches of the laminates, and this brings about a significant decrease in the searching time. The branch and bound processes are performed until the innermost ply is reached. This gives the optimal laminate deterministically.

6. OPTIMIZATION PROCESS OF MULTIPLE LAMINATES

For the optimizations of multiple laminates, an objective function f can be expressed using the lamination parameters of each laminate as follows.

$$f = f(\mathbf{V}_p, \mathbf{W}_p, \mathbf{V}_s, \mathbf{W}_s), \tag{12}$$

where the subscript p means panel and the subscript s means stiffener.

6.1. Tree structure of multiple laminates

First we have to make a tree structure of the multiple laminates. Each laminate of a panel and a stiffener has a tree structure as with a single laminate, as shown in Fig. 5. For laminated composites, the outer plies have a larger effect on the bending stiffness than on the inner plies. For the effective pruning of branches in the branch and bound method, we select variables that have a larger effect on the objective function. In the optimization of multiple laminates, therefore, we should determine the outer plies first of all in both the laminate of the panel and the laminate of the stiffener.

This suggests a tree structure of the multiple laminates by means of alternating the placement of the plies of the panel and the stiffener, as shown in Fig. 6. For example, the outermost ply of the panel is determined first and the outermost ply of the stiffener is determined next.

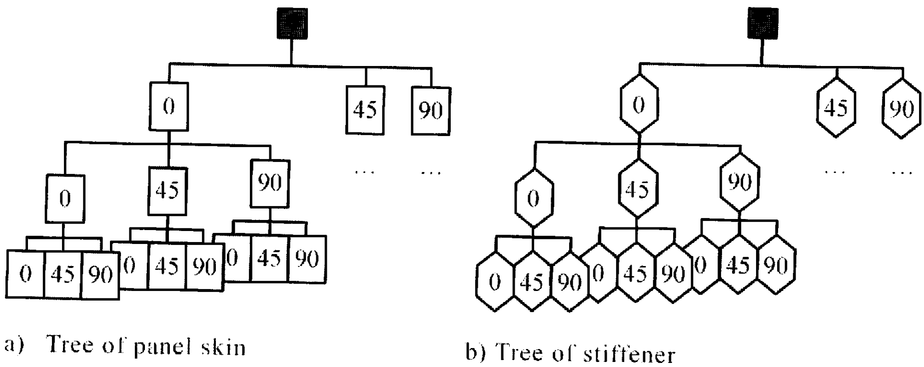


Figure 5. Tree structures of each stacking sequence. (a) Tree of a panel skin. (b) Tree of a stiffener.

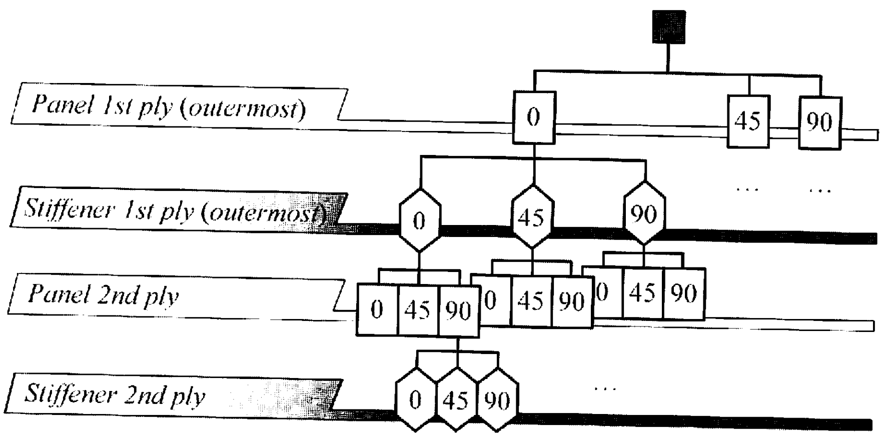


Figure 6. Tree structure of multiple stacking sequences.

6.2. Estimation function of optimizations of multiple laminates

In the next process, we need to consider the estimation function g. Let us consider the case in which the outer plies are determined in both the panel and the stiffener as follows:

$$\begin{aligned}&\text{Panel}: \left[\theta_{1p}/\theta_{2p}/\cdots/\theta_{dp}/*/\cdots/*\right]_S\\&\text{Stiffener}: [\theta_{1s}/\theta_{2s}/\cdots/\theta_{ds}/*/\cdots/*]_S\,.\end{aligned} \tag{13}$$

The lamination parameters corresponding to this branch are obtained from equation (7).

$$\mathbf{V}_p = \mathbf{V}_{0p} + a_{dp}^{V}\mathbf{V}'_p, \quad \mathbf{W}_p = \mathbf{W}_{0p} + a_{dp}^{W}\mathbf{W}'_p, \tag{14}$$

$$\mathbf{V}_s = \mathbf{V}_{0s} + a_{ds}^{V}\mathbf{V}'_s, \quad \mathbf{W}_s = \mathbf{W}_{0s} + a_{ds}^{W}\mathbf{W}'_s. \tag{15}$$

Substituting equations (14) and (15) into equation (12) gives an objective function as follows:

$$\begin{aligned}f &= f(\mathbf{V}_p, \mathbf{W}_p, \mathbf{V}_s, \mathbf{W}_s)\\&= f\left(\mathbf{V}_{0p} + a_{dp}^{V}\mathbf{V}'_p, \mathbf{W}_{0p} + a_{dp}^{W}\mathbf{W}'_p, \mathbf{V}_{0s} + a_{ds}^{V}\mathbf{V}'_s, \mathbf{W}_{0s} + a_{ds}^{W}\mathbf{W}'_s\right)\\&= f_0 + f_{V'p} + f_{W'p} + f_{V's} + f_{W's} + f_{\text{int}},\end{aligned} \tag{16}$$

where f_0 is a constant, $f_{V'p}$, $f_{W'p}$, $f_{V's}$ and $f_{W's}$ are the quadratic polynomials of each lamination parameter, and f_{int} is the interaction term of each parameter. This equation suggests that we can estimate conservatively the maximum value of f in the same way as in equation (11) as follows:

$$g = f_0 + \max(f_{V'p}) + \max(f_{W'p}) + \max(f_{V's}) + \max(f_{W's}) + \max(f_{\text{int}}). \tag{17}$$

Each term in equation (11) can be obtained by means of an identical method as described in equation (11). The later searching process of the modified fractal branch and bound method is exactly the same as in the previous fractal branch and bound method.

7. OPTIMIZATION RESULTS AND DISCUSSION

The proposed modified fractal branch and bound method is applied to the buckling load maximization problem of the hat-stiffened panel shown in Fig. 1.

Before the optimization process, we need to construct a response surface of the buckling load with the variables of the lamination parameters. Each laminate has four lamination parameters, and a quadratic polynomial is used here to make a response surface. This suggests that the total number of unknown coefficients of the response surface of the buckling load is 45. Empirically, we need almost twice as many unknown coefficients to obtain the unknown coefficients by means of the least square errors method. Thus, we should prepare 90 sets of data of the buckling load. The D-optimal design of the experiments is performed to obtain the 90 sets of data from the feasible laminates [7]. In addition to the 90 sets, the origin of the lamination parameter space ($\mathbf{V} = \mathbf{W} = 0$) is added, and FEM analyses are conducted for all 91 cases to obtain the buckling load ratio λ.

The additional point of the origin is to reduce the bias at the center of the lamination parameter space, and the laminate of the origin is added twice in the sets. An origin means a complete quasi-isotropic laminate, even for bending. To improve the fitness of the response surface approximation around the maximum point, the top and second laminate are added to the set twice. In total, 97 sets of FEM analyses are prepared to make the response surface buckling load ratio.

The obtained response surface is given below. In this equation, the replacements of the variables and response are identical to those described in Section 4.

$$\begin{aligned} y = {} & 4.451 - 1.987x_1 - 1.060x_2 - 0.6633x_4 + 2.342x_5 + 0.4612x_6 - 0.2378x_8 \\ & + 2.810x_1^2 + 0.5904x_1x_2 - 2.314x_1x_3 - 1.163x_1x_5 + 0.3706x_1x_7 \\ & + 0.4702x_1x_8 - 0.4692x_2x_3 + 0.5032x_2x_4 + 0.4337x_3x_4 - 0.4725x_4^2 \\ & + 0.3534x_4x_8 - 1.434_5x_6 - 1.196x_5x_7 + 0.5097x_6x_7 - 0.9049x_6x_8 \\ & - 0.7577x_7x_8 + 0.4271x_8^2. \end{aligned} \tag{18}$$

The adjusted coefficient of determination is $R^2_{adj} = 0.903$, and the fitness seems good.

After we obtain the response surface, we can perform an optimization of the two stacking sequences; the panel and the stiffener. The obtained optimal laminates are shown in Table 1. To verify the optimality of the results, we conducted many FEM analyses and obtained true optimal laminates. The error in the obtained optimal laminates is only 2.3% and the value is acceptable for practical structures.

Since this modified fractal branch and bound method is a deterministic method, the error in the optimal laminate from the true optimal laminate simply comes from the fitness error of the response surface. Thus the improvement in the fitness around the maximum point provides a better result. To achieve an improvement, we need to reconstruct the response surface by adding several FEM analyses around the maximum point. Our previous studies suggest that a practical optimal result of 0.5% error can be obtained by means of the optimality check method [8] and the modified response surface [16]. The result using this modification is shown in

Table 1.
Optimal stacking sequences using response surface

Stacking sequences		RS val.	FEM val.
Panel skin	$[\pm 45/45/90_2/-45/90_2]_S$	8.123	8.312
Stiffener	$[\pm 45/45/0_2/-45/0_2]_S$		

Table 2.
Optimal stacking sequences with sufficient optimality

Stacking sequences		FEM val.
Panel skin	$[(\pm45)_4]_S$	8.816
Stiffener	$[(\pm45)_2/0/90/0_2]_S$	

Table 3.
Buckling load factor of quasi-isotropic laminates

Stacking sequences		FEM val.
Panel skin	$[45/0/90/-45/90/-45/45/0]_S$	4.253
Stiffener	$[45/0/90/-45/90/-45/45/0]_S$	

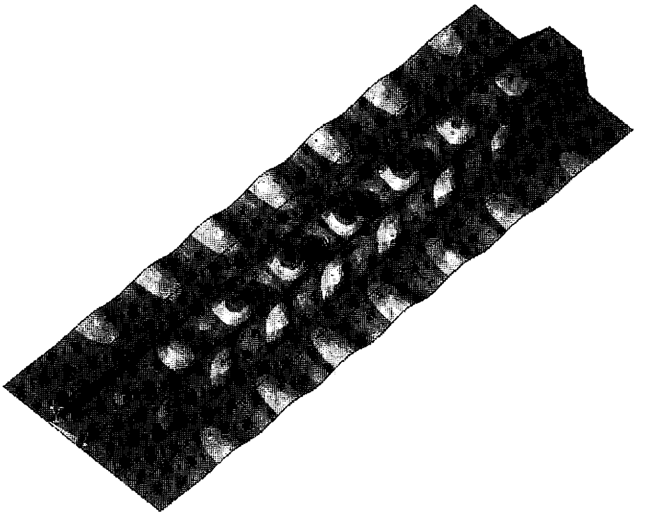


Figure 7. Buckling mode shape of the optimum.

Table 2. The total number of FEM analyses required is approximately 100, and the difference in the result from the optimal laminate of the modified fractal branch and bound method in Table 1 is only 5.7%. This means we do not need additional FEM analyses to improve the fitness around the maximum point.

The buckling mode of the stiffened panel is shown in Fig. 7. As a reference, the buckling load and buckling mode of the quasi-isotropic laminates are shown in Table 3 and Fig. 8, respectively. In the optimal laminates, the buckling load is improved 90% compared with that of the quasi-isotropic laminates, and buckling in both the panel and stiffener is observed in the optimal laminates.

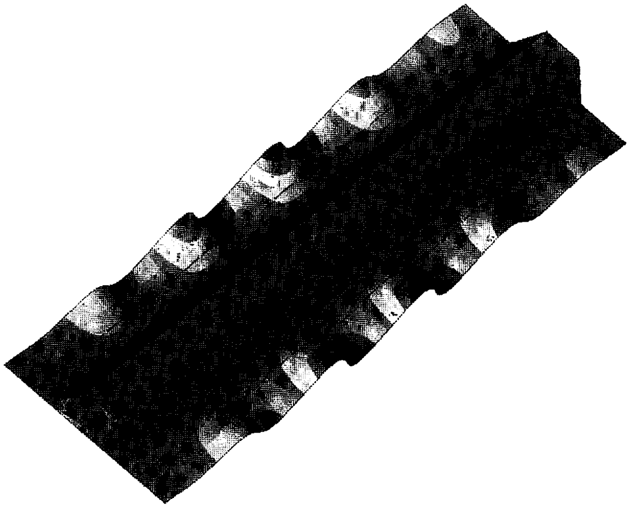


Figure 8. Buckling mode shape of quasi-isotropic laminates.

8. CONCLUDING REMARKS

In the present study, an optimization of multiple stacking sequences such as in stiffened panels is discussed. To modify the conventional fractal branch and bound method that is useful only for the optimization of a single stacking sequence, a tree structure and estimation function of the optimizations of multiple stacking sequences is first proposed. A modification of the fractal branch and bound method is proposed here. This modified fractal branch and bound method is applied to a hat-stiffened panel and the optimization of the stacking sequences of a panel and stiffener is performed here as a feasibility study. As a result, the effectiveness of the method is shown. It is concluded that the additional modified response surface is not required when an error of 5% in practice is permitted in the problem.

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